

Homework No. 04 (Spring 2015)

PHYS 530A: Quantum Mechanics II

Due date: None

1. (30 points.) The Pauli matrices satisfy

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk} \sigma_k. \quad (1)$$

A particular representation of Pauli matrices is

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2)$$

- (a) Construct the matrix

$$\sigma_{\theta,\phi} = \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}(\theta, \phi) = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}, \quad (3)$$

where

$$\boldsymbol{\sigma} = \sigma_x \hat{\mathbf{i}} + \sigma_y \hat{\mathbf{j}} + \sigma_z \hat{\mathbf{k}}, \quad (4)$$

$$\hat{\mathbf{n}}(\theta, \phi) = \sin \theta \cos \phi \hat{\mathbf{i}} + \sin \theta \sin \phi \hat{\mathbf{j}} + \cos \theta \hat{\mathbf{k}}. \quad (5)$$

Find the eigenvalues $\sigma'_{\theta,\phi}$ and the normalized eigenvectors, $|\sigma'_{\theta,\phi} = +1\rangle$ and $|\sigma'_{\theta,\phi} = -1\rangle$, (up to a phase) of the matrix $\sigma_{\theta,\phi}$.

- (b) Now compute the matrices

$$\bar{\sigma}_x = \begin{pmatrix} \langle \sigma'_{\theta,\phi} = +1 | \sigma_x | \sigma'_{\theta,\phi} = +1 \rangle & \langle \sigma'_{\theta,\phi} = +1 | \sigma_x | \sigma'_{\theta,\phi} = -1 \rangle \\ \langle \sigma'_{\theta,\phi} = -1 | \sigma_x | \sigma'_{\theta,\phi} = +1 \rangle & \langle \sigma'_{\theta,\phi} = -1 | \sigma_x | \sigma'_{\theta,\phi} = -1 \rangle \end{pmatrix}, \quad (6)$$

$$\bar{\sigma}_y = \begin{pmatrix} \langle \sigma'_{\theta,\phi} = +1 | \sigma_y | \sigma'_{\theta,\phi} = +1 \rangle & \langle \sigma'_{\theta,\phi} = +1 | \sigma_y | \sigma'_{\theta,\phi} = -1 \rangle \\ \langle \sigma'_{\theta,\phi} = -1 | \sigma_y | \sigma'_{\theta,\phi} = +1 \rangle & \langle \sigma'_{\theta,\phi} = -1 | \sigma_y | \sigma'_{\theta,\phi} = -1 \rangle \end{pmatrix}, \quad (7)$$

$$\bar{\sigma}_z = \begin{pmatrix} \langle \sigma'_{\theta,\phi} = +1 | \sigma_z | \sigma'_{\theta,\phi} = +1 \rangle & \langle \sigma'_{\theta,\phi} = +1 | \sigma_z | \sigma'_{\theta,\phi} = -1 \rangle \\ \langle \sigma'_{\theta,\phi} = -1 | \sigma_z | \sigma'_{\theta,\phi} = +1 \rangle & \langle \sigma'_{\theta,\phi} = -1 | \sigma_z | \sigma'_{\theta,\phi} = -1 \rangle \end{pmatrix}. \quad (8)$$

These matrices are representation of the Pauli matrices in the eigenbasis of $\sigma_{\theta,\phi}$.

- (c) Show that

$$\bar{\sigma}_i \bar{\sigma}_j = \delta_{ij} + i\varepsilon_{ijk} \bar{\sigma}_k. \quad (9)$$