

Homework No. 07 (Spring 2015)

PHYS 530A: Quantum Mechanics II

Due date: Tuesday, 2015 Apr 21, 4.30pm

1. **(20 points.)** We constructed the total angular momentum states of two spin- $\frac{1}{2}$ systems, $j_1 = \frac{1}{2}$, $j_2 = \frac{1}{2}$, by beginning with the total angular momentum state

$$|1, 1\rangle = \left| \frac{1}{2}, \frac{1}{2} \right\rangle_{\textcircled{1}} \left| \frac{1}{2}, \frac{1}{2} \right\rangle_{\textcircled{2}} \quad (1)$$

and using the lowering operator to construct the $|1, 0\rangle$ and $|1, -1\rangle$ states. The state $|0, 0\rangle$ was then constructed (to within a phase factor) as the state orthogonal to $|1, 0\rangle$.

- (a) Repeat this exercise by beginning with the total angular momentum state $|1, -1\rangle$ and using the raising operator to construct $|1, 0\rangle$ and $|1, 1\rangle$ states.
- (b) Investigate the property of the total angular momentum states under the interchange $\textcircled{1} \leftrightarrow \textcircled{2}$. In particular, find out if each of the total angular momentum states are symmetrical (do not change sign) or antisymmetrical (change sign).
2. **(40 points.)** Let us construct the total angular momentum states for the composite system built out of two angular momenta $j_1 = 2$, $j_2 = \frac{1}{2}$.

- (a) Determine the total number of states by counting the individual states,

$$\left(\sum_{m_1=-j_1}^{j_1} \right) \left(\sum_{m_2=-j_2}^{j_2} \right). \quad (2)$$

Repeat this by counting the number of total angular momentum states,

$$\sum_{j=|j_1-j_2|}^{j_1+j_2} \sum_{m=-j}^j. \quad (3)$$

- (b) Beginning with $|5/2, 5/2\rangle$ use the lowering operator to build five other states with $j = 5/2$.
- (c) Construct $|3/2, 3/2\rangle$ state by requiring it to be orthogonal to $|5/2, 3/2\rangle$, and be normalized.
- (d) Beginning with $|3/2, 3/2\rangle$ use the lowering operator to build three other states with $j = 3/2$.

Particle	$ T, T_3\rangle$	Q
proton	$ \frac{1}{2}, \frac{1}{2}\rangle$	1
neutron	$ \frac{1}{2}, -\frac{1}{2}\rangle$	0
π^+	$ 1, 1\rangle$	1
π^0	$ 1, 0\rangle$	0
π^-	$ 1, -1\rangle$	-1

Table 1: Isospin assignments for particles.

3. **(60 points.)** Construct the total angular momentum state $|1, 1\rangle$ for the composite system built out of two angular momenta $j_1 = 2, j_2 = 1$. Investigate the property of this state under the interchange ① $\leftrightarrow$ ②.
4. **(20 points.)** (Schwinger's QM book, Prob. 3-4a.) Iso(topic) spin T : The nucleon is a particle of isospin $T = \frac{1}{2}$; the state with $T_3 = \frac{1}{2}$ is the proton (p), the state with $T_3 = -\frac{1}{2}$ is the neutron (n). Electric charge of a nucleon is given by $Q = \frac{1}{2} + T_3$. The π meson, or pion, has isospin $T = 1$, and electric charge $Q = T_3$, so there are three kinds of pions with different electric charge: $T_3 = 1$ (π^+), $T_3 = 0$ (π^0), $T_3 = -1$ (π^-). (Refer Table 1.)

Consider the system of a nucleon and a pion. The electric charge of this system is $Q = \frac{1}{2} + T_3$. Check that a system of charge 2, $T_3 = \frac{3}{2}$, is $p + \pi^+$, according to the isospin assignments. Now, if the system is in the state $T = \frac{3}{2}, T_3 = \frac{1}{2}$, what is the probability of finding a proton? What is the accompanying π -meson?