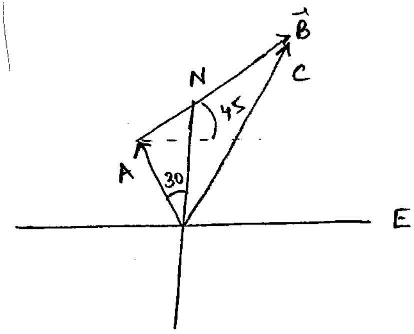
Solutions to Final Exam (PHYS 203A, Summer 2015)

prob. 1



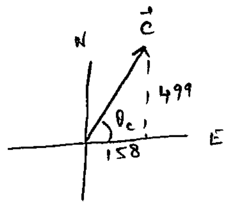
$\vec{A} = -250\sin 30\,\hat{i} + 250\cos 30\,\hat{j}$

$\vec{B} = +400\cos 45\,\hat{i} + 400\sin 45\,\hat{j}$

$\vec{C} = \vec{A} + \vec{B}$

$= (-250\sin 30 + 400\cos 45)\,\hat{i} + (250\cos 30 + 400\sin 45)\,\hat{j}$

$= 158\,\hat{i} + 499\,\hat{j}$



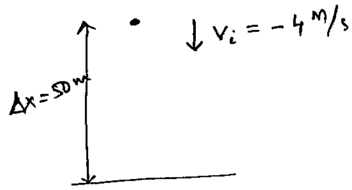
$|\vec{C}| = \sqrt{158^2 + 499^2} = 523\text{ m}$

$\theta_c = \tan^{-1}\left(\frac{499}{158}\right) = 72.4^\circ$ (North of East)

prob. 2

$V_i = -4\text{ m/s}$ $\quad \Delta x = -50.0\text{m}$ $\quad a = -9.80\text{ m/s}^2$.

$V_f =$ $\quad \Delta t =$



$\Delta x = V_i\,\Delta t + \frac{1}{2} a \Delta t^2$.

$-50 = -4\,\Delta t - \frac{1}{2} 9.8\,\Delta t^2$

$4.9\,\Delta t^2 + 4\,\Delta t - 50 = 0$

$\Delta t = \frac{-4 \pm \sqrt{4^2 - 4(4.9)(-50)}}{2 \times 4.9}$

$= \frac{-4 \pm 31.6}{9.8}$

$= +2.82\text{ sec}$ or -3.68 sec.

Since the fish hits the water in future the correct answer is $+2.82$ sec.

prob. 3

x-dir

$\Delta x = 30.0\,m$

$\Delta t =$

$V_{ix} = \dfrac{\Delta x}{\Delta t}$.

y-dir

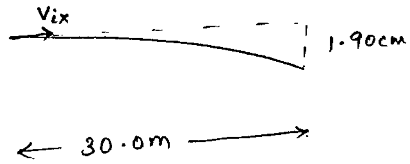
$\Delta y = -1.90\,cm$

$\Delta t =$

$V_{iy} = 0$

$V_{fy} =$

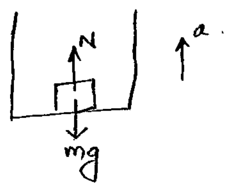
$a = -9.80\,m/s^2$.



$$\Delta y = \cancel{V_{iy}}^{=0}\,\Delta t + \tfrac{1}{2} a \Delta t^2 \quad \Rightarrow \quad \Delta t = \sqrt{\frac{2\Delta y}{a}}$$

$$V_{ix} = \frac{\Delta x}{\sqrt{\frac{2\Delta y}{a}}} = \frac{30.0}{\sqrt{\frac{2\times 1.90\times 10^{-2}}{9.80}}} = 482\ \frac{m}{s}$$

prob. 4



(a) a is downward

$$N - mg = -ma.$$

$$N = m(g-a) = 75.0 \times (9.80 - 2.00)$$

$$= 585\ N.$$

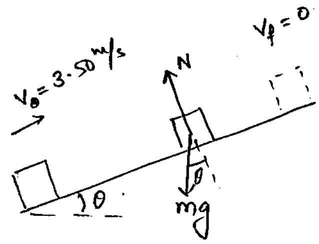
(b) a is downward

$$N - mg = -ma \quad \Rightarrow \quad N = 585\,N.$$

(c) a is zero

$$N = mg = 75.0 \times 9.80 = 735\ N.$$

prob. 5



$$\vec{N} + m\vec{g} = m\vec{a}$$

x: $0 - mg\sin\theta = ma$

$\Rightarrow \quad a = -g\sin\theta$

y: $N - mg\cos\theta = 0$

$\Rightarrow \quad N = mg\cos\theta$

$V_i = 3.50\,\text{m/s}$ $\quad \Delta x = ?$ $\quad a = -g\sin\theta$

$V_f = 0$ $\quad \Delta t =$ $\quad = -9.80\sin 30$

$= -4.90\,\text{m/s}^2$

$$\Delta x = \frac{V_f^2 - V_i^2}{2a} = \frac{0 - 3.50^2}{2\times(-4.90)} = 1.25\,\text{m}$$

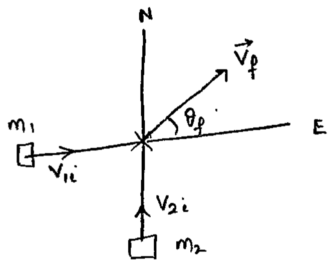
prob. 6

$$K_A + U_A = K_E + U_E$$

$$\frac{1}{2} m \underbrace{V_A^2}_{=0} + \cancel{m}gh_A = \frac{1}{2}\cancel{m}V_E^2 + \cancel{m}gh_E$$

$$V_E = \sqrt{2g(h_A - h_E)} = \sqrt{2\times 9.80\times(50.0-20.0)} = 24.2\,\text{m/s}$$

prob. 7



$$(m_1 + m_2)\vec{V_f} = m_1\vec{V_{1i}} + m_2\vec{V_{2i}}$$

$$(6000. + 2500)\vec{V_f} = 2500\times 20.0\,\hat{i} + 6000\times 25.0\,\hat{j}$$

$$\vec{V_f} = \frac{25}{85}\times 20.0\,\hat{i} + \frac{60}{85}\times 25.0\,\hat{j}$$

$$= 5.88\,\hat{i} + 17.7\,\hat{j}$$

$$|\vec{V_f}| = \sqrt{5.88^2 + 17.7^2} = 18.7\,\frac{\text{m}}{\text{s}}$$

$$\theta_f = \tan^{-1}\left(\frac{17.7}{5.88}\right) = 71.6^\circ$$ (North of East)

prob. 8

$\omega_i = ?$ $\qquad \alpha = 0.170 \frac{rad}{s^2}$ $\qquad \Delta\theta = 2839 \text{ rev} \frac{2\pi \text{ rad}}{1 \text{ rev}}$

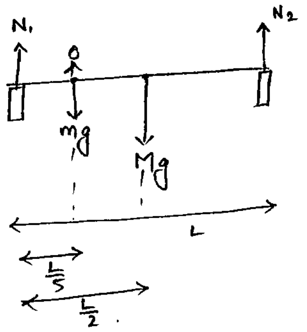
$\omega_f = 132 \frac{rad}{s}$ $\qquad \Delta t =$ $\qquad = 17840 \text{ rad}$

$$2\alpha\Delta\theta = \omega_f^2 - \omega_i^2$$

$$\omega_i^2 = \omega_f^2 - 2\alpha\Delta\theta$$

$$\omega_i = \sqrt{132^2 - 2 \times 0.170 \times 17840} = 106.6 \frac{rad}{s}$$

prob. 9



m - person
M - bridge

Force:

$$N_1 + N_2 - Mg - mg = 0$$

$$\Rightarrow \quad N_1 + N_2 = 4460 \quad \text{— (i)}$$

Torque (about center)

$$-N_1 \frac{L}{2} + N_2 \frac{L}{2} + 0 + mg \frac{3}{10} L = 0$$

$\frac{L}{2} - \frac{L}{5} = \frac{3}{10}$

$$\Rightarrow \quad -N_1 + N_2 + \frac{3}{5} \times 875 = 0$$

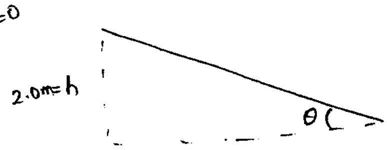
$$\Rightarrow \quad N_1 - N_2 = 525 \quad \text{— (ii)}$$

Solving (i) and (ii)

$$N_1 = 2490 \text{ N}$$

$$N_2 = 1970 \text{ N}$$

prob. 10



$$\frac{1}{2} m \cancel{v_i^2}^{=0} + \frac{1}{2} I \cancel{\omega_i^2}^{=0} + mgh_i = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + mg\cancel{h_f}^{=0}$$

$$\cancel{m} g h_i = \underbrace{\left(\frac{1}{2} + \frac{1}{5}\right)}_{\frac{7}{10}} \cancel{m} v_f^2$$

$$v_f = \sqrt{\frac{10}{7} g h_i} = \sqrt{\frac{10}{7} \times 9.8 \times 2.00} = 5.29 \frac{m}{s}$$

prob. 11

$$L_i^{disc} + L_i^{kid} = L_f^{disc} + L_f^{kid}$$

$$I_i^{disc}\,\omega_i^{disc} + I_i^{kid}\,\omega_i^{kid} = I_f^{disc}\,\omega_f^{disc} + I_f^{kid}\,\omega_f^{kid}.$$

$\omega_i^{disc} = \omega_i^{kid}$
$\omega_f^{disc} = \omega_f^{kid}$

$$\left(\tfrac{1}{2} MR^2 + mR^2\right)\omega_i = \left(\tfrac{1}{2} MR^2 + m\times 0^2\right)\omega_f.$$

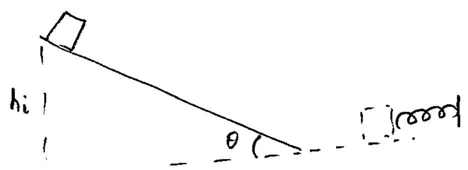
$$\omega_f = \left(\frac{\frac{1}{2}M\cancel{R^2} + m\cancel{R^2}}{\frac{1}{2}M\cancel{R^2}}\right)\omega_i$$

$$= \left(1 + \frac{2m}{M}\right)\omega_i = \left(1 + \frac{2\times 25.0}{100.0}\right)\times 2.10\ \frac{rad}{s}$$

$$= 3.15\ \frac{rad}{s}$$

prob. 12

$$\tfrac{1}{2}m\underbrace{v_i^2}_{=0} + mgh_i + \tfrac{1}{2}k\underbrace{x_i^2}_{=0} = \tfrac{1}{2}m\underbrace{v_f^2}_{=0} + mg\underbrace{h_f}_{=0} + \tfrac{1}{2}kx_f^2.$$



$$mgh_i = \tfrac{1}{2}kx_f^2.$$

$$1.00\times 9.80\times 2.00 = \tfrac{1}{2}\,2000.0\times x_f^2.$$

$$x_f = 0.14\,m = 14\,cm$$