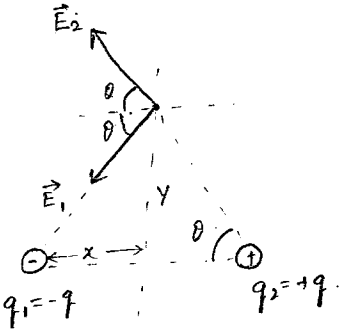
Solutions to Final Exam, PHYS-205B, Fall 2015

Prob. 1



$$\vec{E}_1 = -E_1 \cos\theta\, \hat{i} - E_1 \sin\theta\, \hat{j}$$

$$\vec{E}_2 = -E_2 \cos\theta\, \hat{i} + E_2 \sin\theta\, \hat{j}$$

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$$

$$= -2E_1 \cos\theta\, \hat{i}$$

$$= -2 \times 1.24 \times 10^4 \frac{N}{C} \frac{1.00}{\sqrt{1.00^2 + 2.00^2}}$$

$$= -9.2 \times 10^3 \frac{N}{C}$$

$$E_1 = E_2 = \frac{kq}{x^2+y^2}$$

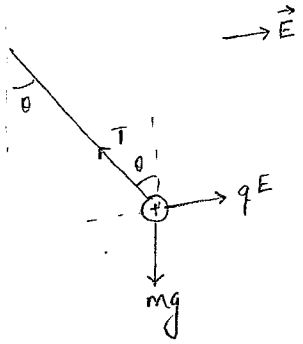
$$= \frac{9.0\times10^9 \times 1.0\times10^{-9} C}{(1.00^2 + 2.00^2)\times10^{-4} m^2}$$

$$= 1.24 \times 10^4 \frac{N}{C}$$

Magnitude $= 9.2 \times 10^3 \frac{N}{C}$

Direction : $-\hat{i}$

Prob. 2



$$T \sin\theta = qE$$

$$T \cos\theta = mg$$

$$\tan\theta = \frac{qE}{mg}$$

$$\theta = \tan^{-1}\left(\frac{qE}{mg}\right)$$

$$= \tan^{-1}\left(\frac{10.0\times10^{-6} C \times 1.0\times10^3 \frac{N}{C}}{1.0\times10^{-3} kg \times 10.0 \frac{m}{s^2}}\right)$$

$$= \tan^{-1}(1.0)$$

$$= 45^\circ$$

Prob. 3

$$\vec{E} = -\hat{i}\frac{\partial}{\partial x}V - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$$

$$\frac{\partial V}{\partial x} = b \qquad \frac{\partial V}{\partial y} = 0 \qquad \frac{\partial V}{\partial z} = 0$$

$$= -\hat{i}\, b$$

$$= +\hat{i}\ 450\ \frac{V}{cm}$$

$$= \hat{i}\ 4.50\times 10^{4}\ \frac{V}{m}$$

Magnitude : $4.5\times10^{4}\ \frac{V}{m}$

Direction : $\hat{i}$

Prob. 4

(a) $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10.0} + \frac{1}{20.00} \Rightarrow C_{eq} = 6.67\,\mu F$

(b) $Q_1 = Q_2 = C_{eq} V = 6.67\times10^{-6}\times 10 = 66.7\,\mu C$

(c) $V_1 = \frac{Q_1}{C_1} = \frac{66.7\,\mu C}{10.0\,\mu F} = 6.67\ V$

$V_2 = \frac{Q_2}{C_2} = \frac{66.7\,\mu C}{20.0\,\mu F} = 3.33\ V$

(d) $U_1 = \frac{Q_1^2}{2C_1} = \frac{(66.7\,\mu C)^2}{2\times 10.0\,\mu F} = 0.222\ mJ$

$U_2 = \frac{Q_2^2}{2C_2} = \frac{(66.7\,\mu C)^2}{2\times 20.0\,\mu F} = 0.111\ mJ$

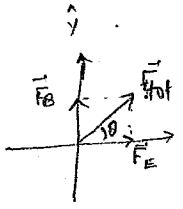
Prob. 5

$\vec{F}_E = q\vec{E} = \hat{x}\ 1.0\times10^{-6} C \times 3.00\times10^{3}\ \frac{N}{C}$

$= \hat{x}\ 3.0\ mN$

$\vec{F}_B = q\vec{v}\times\vec{B} = \hat{y}\ 1.0\times10^{-6} C \times 2.0\times10^{6}\ \frac{m}{s} \times 2.00\times10^{-3} T \sin 90$

$= \hat{y}\ 4.0\ mN$

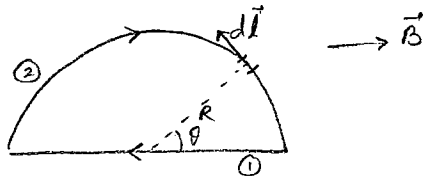


$\vec{F}_{tot} = \hat{x}\ 3.0\ mN + \hat{y}\ 4.0\ mN$

$|\vec{F}_{tot}| = \sqrt{3.0^2 + 4.0^2}\ mN = 5.0\ mN$ → magnitude

$\theta = \tan^{-1}\left(\frac{4.0}{3.0}\right) = 53.1^\circ$ → direction clockwise w.r.t $\hat{i}$.

Prob. 6



(a) $\vec{F}_2 = \int_{②} I\, d\vec{l} \times \vec{B}$

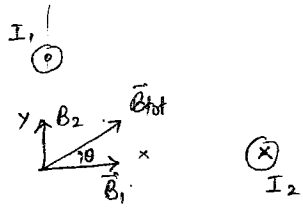
$= I \int_0^{\pi} R\, d\theta\, B \sin(90+\theta)$

$= I R B \underbrace{\int_0^{\pi} d\theta \cos\theta}_{=0} = 0$

(b) $\vec{F}_1 = I \vec{L} \times \vec{B} = I\, 2R B \underbrace{\sin 180}_{=0} = 0$

(c) $\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 = 0$

Prob. 7



$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi y}\hat{i} = \hat{i}\,\frac{4\pi\times10^{-7}\times1.0}{2\pi\times3.0\times10^{-2}} = \hat{i}\,6.67\,\mu T$

$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi x}\hat{j} = \hat{j}\,\frac{4\pi\times10^{-7}\times2.0}{2\pi\times4.0\times10^{-2}} = \hat{j}\,10.0\,\mu T$

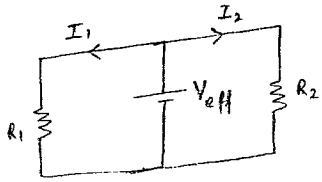
$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 = \hat{i}\,6.67\mu T + \hat{j}\,10.0\mu T$

$|\vec{B}_{tot}| = \sqrt{6.67^2 + 10.0^2}\,\mu T = 12\,\mu T \longrightarrow$ magnitude

$\theta = \tan^{-1}\left(\frac{10.0}{6.67}\right) = 56.3^\circ \longrightarrow$ direction clockwise w.r.t $\hat{i}$

Prob. 8

effective circuit



$V_{eff} = B l v$

$= 0.20 \times 10.0\times10^{-2} \times 10.0\ \frac{m}{s}$

$= 0.20\ V$

$I_1 = \frac{V_{eff}}{R_1} = \frac{0.20}{100.0} = 2.0\ mA$

$I_2 = \frac{V_{eff}}{R_2} = \frac{0.20}{200.0} = 1.0\ mA$

Prob. 9

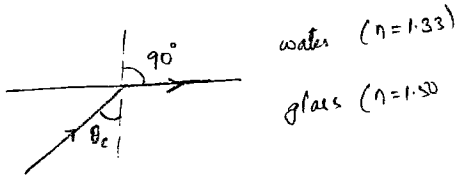
RL circuit: $\tau = \frac{L}{R}$ Given $\frac{L}{R} = RC$

RC circuit: $\tau = RC$ $\Rightarrow$ $R = \sqrt{\frac{L}{C}}$

(a) $R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00\ H}{2.00\times10^{-6}\ F}} = 1.23\ M\Omega$

(b) $\tau = RC = 1.23\times10^{3} \times 2.00\times10^{-6} = 2.46\ \mu s$

Prob. 10



$n_g \sin\theta_c = n_w \underbrace{\sin 90}_{=1}$

$\theta_c = \sin^{-1}\left(\frac{1.33}{1.50}\right) = 62.5°$

Prob. 11

(a) $R = 2f = 20.0\ cm$

(b) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{q} = \frac{1}{10.0} - \frac{1}{15.0} = \frac{1}{30}$

$\Rightarrow q = 30.0\ cm$

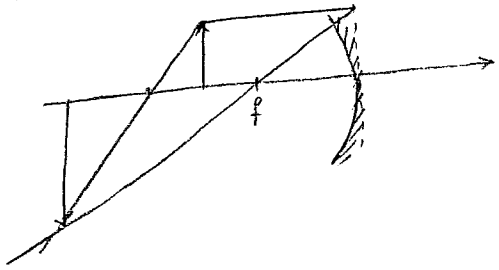
(c) $m = -\frac{q}{p} = -2$

(d) Real.

(e) Inverted

(f) $h_i = m h_o = -2 \times 1.0\ cm = -2.0\ cm$

(f)



Prob. 12

(a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{q} = \frac{1}{10.0} - \frac{1}{10.0}$

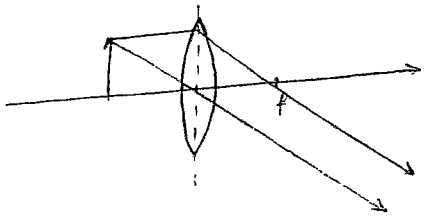
$\Rightarrow q \to \infty$

(b) $m = -\frac{q}{p} \to -\infty$

(c) real

(d) inverted

(f)



Note: we assumed $p = (10.0 - \delta)\ cm$ with $\delta \to 0$.