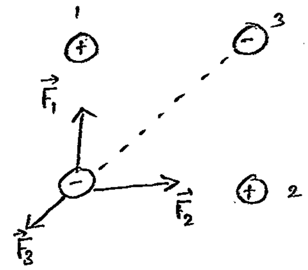
Solutions to Midterm Exam 01, Spring 2016

Prob. 1



(a)

$$\vec{F}_1 = 0\,\hat{i} + F_0\,\hat{j}$$

$$\vec{F}_2 = F_0\,\hat{i} + 0\,\hat{j}$$

$$\vec{F}_3 = -\frac{F_0}{2}\cos 45\,\hat{i} - \frac{F_0}{2}\sin 45\,\hat{j}$$

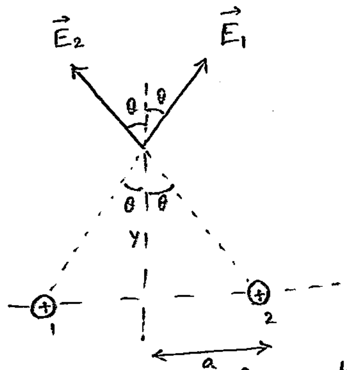
$$F_0 = \frac{kQ^2}{L^2}$$

$$\vec{F}_{tot} = F_0\left(1-\frac{1}{2\sqrt{2}}\right)\hat{i} + F_0\left(1-\frac{1}{2\sqrt{2}}\right)\hat{j}$$

$$|\vec{F}_{tot}| = \sqrt{2}\,F_0\left(1-\frac{1}{2\sqrt{2}}\right) = \frac{kQ^2}{L^2}\left(\sqrt{2}-\frac{1}{2}\right)$$

(b) Since each component of $\vec{F}_{tot}$ is positive, because $1-\frac{1}{2\sqrt{2}} > 0$, we conclude that the resultant force on each charge is towards the center of square. Thus, the charges move towards each other.

Prob. 2



$E_1 \sin$

$$\vec{E}_1 = +\frac{kQ}{y^2+a^2}\sin\theta\,\hat{i} + \frac{kQ}{y^2+a^2}\cos\theta\,\hat{j}$$

$$\vec{E}_2 = -\frac{kQ}{y^2+a^2}\sin\theta\,\hat{i} + \frac{kQ}{y^2+a^2}\cos\theta\,\hat{j}$$

$$\vec{E}_{tot} = 0\,\hat{i} + 2\frac{kQ}{y^2+a^2}\cos\theta\,\hat{j}$$

$$= \hat{j}\;2\frac{kQ}{y^2+a^2}\frac{y}{\sqrt{y^2+a^2}}$$

$$= \hat{j}\;\frac{2\times 8.99\times10^9\times 1.0\times10^{-9}\times 2.50\times10^{-2}}{\left[(2.50\times10^{-2})^2 + (1.00\times10^{-2})^2\right]^{3/2}}$$

2.69×10^{-4} $\quad 6.25\times10^{-4}$ $\quad 10^{-4}$ $\quad 6.25\times10^{-4}$

$$= \hat{j}\;\cancel{1.67}\times10^{5}\,\frac{N}{C}$$

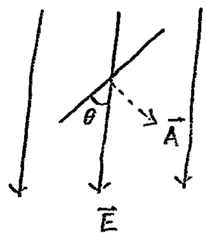
2.30 $\quad 69.52$

$$\frac{44.95\times10^{-2}}{(7.25\times10^{-4})^{3/2}} = \frac{44.95\times10^{-2}}{(2.69\times10^{-2})^{3}} = 2.31\times10^{4}\,\frac{N}{C}.$$

magnitude $= 1.67\times10^{5}\,\frac{N}{C}$.

direction: along $+\hat{j}$

Prob. 5



$$\phi_E = \vec{E} \cdot \vec{A}$$

$$= E A \cos(90-30)$$

$$= 3.0 \times 10^2 \ \frac{N}{C} \times 2.0 \ m^2 \ \overbrace{\cos 60}^{\frac{1}{2}}$$

$$= 3.0 \times 10^2 \ \frac{N m^2}{C}.$$

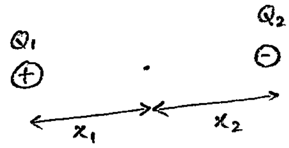
Prob. 6

$$\phi_E = \frac{Q_{en}}{\epsilon_0} = \frac{(+2.80 - 9.00 + 27.0 - 61.2) \times 10^{-6} \ C}{8.85 \times 10^{-12} \ \frac{C^2}{Nm^2}}$$

$$= -4.57 \times 10^6 \ \frac{N m^2}{C}$$

$V = E d$
$= \frac{N}{C} m.$

Prob. 7



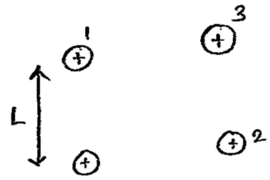
$x = x_1 = x_2$.

$$V = V_1 + V_2$$

$$= \frac{k|Q_1|}{|x_1|} - \frac{k|Q_2|}{|x_2|}$$

$$= \frac{k(|Q_1| - |Q_2|)}{x} = \frac{8.99 \times 10^9 \times (4.0 - 2.0) \times 10^{-9} \ C}{\left(\frac{1.0}{2}\right)} = 35.96 \ \text{Volt}$$

Prob. 8



$$\Delta U = \frac{kq^2}{L} + \frac{kq^2}{L} + \frac{kq^2}{\sqrt{2} L}$$

$$= \left(2 + \frac{1}{\sqrt{2}}\right) \frac{kq^2}{L}$$

$$= \left(2 + \frac{1}{\sqrt{2}}\right) \frac{8.99 \times 10^9 \ (1.00 \times 10^{-6})^2}{(10.0 \times 10^{-2})}$$

$$= 0.243 \ \text{Joules}.$$

Prob. 9

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{E} = -\vec{\nabla} V$$

$$= -\hat{i}\frac{\partial V}{\partial x} + 0\hat{j} + 0\hat{k}$$

$$= -\hat{i}\, b$$

$$= \hat{i}\; 450 \frac{V}{cm}$$

magnitude $= 450 \frac{V}{cm}$

direction = along $+\hat{i}$

Prob. 10

$\vec{E} = 0$ (since it is inside the conductor.)

$V = \frac{kQ}{R}$ (potential at center is same as that on the surface.)

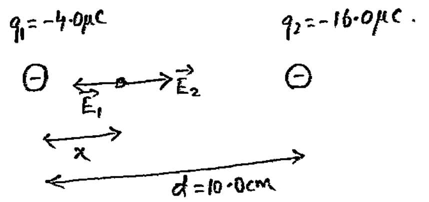
$$= \frac{8.99\times10^{9} \times 1.0\times10^{-6}}{10.0\times 10^{-2} m}$$

$$= 8.99 \times 10^{4} \text{ Volts}$$

$\bar{0}$ ~~#~~ 450 . 2

5

Prob. 3



$$|\vec{E}_1| = |\vec{E}_2|$$

$$\frac{k|q_1|}{x^2} = \frac{k|q_2|}{(d-x)^2}.$$

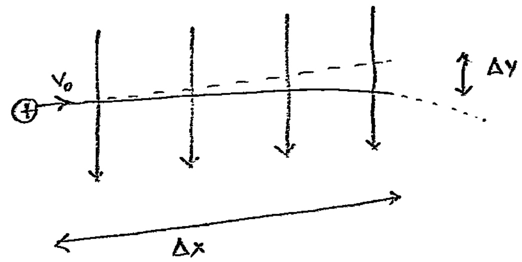
$$d - x = \pm x\sqrt{\frac{|q_2|}{|q_1|}}$$

$$d - x = \pm 2x$$

$$x = \frac{d}{1 \pm 2} = \frac{10.0\text{ cm}}{1 \pm 2} = 3.33\text{cm} \quad \text{or} \quad -10.0\text{cm}$$

$x = 3.33$cm is the correct solution, because we began with the assumption that $x > 0$.

Prob. 4



(a) The proton is not accelerating in the x-direction. Thus,

the

$$\Delta t = \frac{\Delta x}{v_0} = \frac{4.50 \times 10^{-2} m}{4.80 \times 10^5 \frac{m}{s}} = 9.3 \times 10^{-8} s$$

$$a_y = \frac{qE}{m}$$

(b) $$\Delta y = +\frac{1}{2} a_y \Delta t^2.$$

$$= -\frac{1}{2} \frac{qE}{m} \Delta t^2$$

$$= -\frac{1}{2} \frac{1.6 \times 10^{-19} C \times 8.80 \times 10^3}{1.67 \times 10^{-27} kg} \times (9.3 \times 10^{-8})^2$$

$$= -364.6 \times 10^{-5} m.$$

$$= -3.67 \text{ mm}.$$

$$= -7.29 \text{ mm}$$

2.56

3.67