Midterm Exam No. 01 (Spring 2015)

PHYS 530A: Quantum Mechanics II

Date: 2015 Feb 16

1. (**30 points.**) The motion of a particle of mass m undergoing simple harmonic motion is described by

$$\frac{d}{dt}(mv) = -kx, \tag{1}$$

where $v = dx/dt$ is the velocity in the x direction.

   (a) Find the Lagrangian for this system that implies the equation of motion of Eq. (1) using Hamilton's principle of stationary action.

   (b) Determine the canonical momentum for this system

   (c) Determine the Hamilton $H(p, x)$ for this system.

2. (**40 points.**) Harmonic oscillations are described by the Hamiltonian

$$H(x,p) = \frac{1}{2}p^2 + \frac{1}{2}x^2. \tag{2}$$

   (a) Determine the equations of motions using

$$\frac{dx}{dt} = \frac{\partial H}{\partial p} \qquad \text{and} \qquad \frac{dp}{dt} = -\frac{\partial H}{\partial x}. \tag{3}$$

   Then, solve the coupled differential equations to find the solution

$$x = x_0 \cos t + p_0 \sin t, \tag{4}$$

   where x_0 and p_0 are given using the intial conditions at $t = 0$.

   (b) Next, determine the equations of motion using

$$\big[x, H\big] = \frac{\partial H}{\partial p} \qquad \text{and} \qquad \big[p, H\big] = -\frac{\partial H}{\partial x}. \tag{5}$$

   Show that

$$\big[\ldots\big[\big[x,H\big],H\big],\ldots\big] = \begin{cases} \frac{1}{i} i^N p, & \text{for number of commutators, } N\text{, being odd,} \\ i^N x, & \text{for number of commutators, } N\text{, being even.} \end{cases} \tag{6}$$

   Then, using

$$x = x_0 + t\big[x,H\big]_0 + \frac{1}{2}t^2\big[\big[x,H\big],H\big]_0 + \cdots \tag{7}$$

   rederive the solution. Here the subscript zero refers to the initial conditions at $t = 0$.

3. (**30 points.**) Consider the matrix

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}. \tag{8}$$

(a) Find all the eigenvalues of the matrix A.

(b) Find the normalized eigenvectors associated with all the eigenvalues of matrix A. (Simplification is achieved by writing the trignometric functions in terms of half angles. $1 - \cos\theta = 2\sin^2\theta/2$, $1 + \cos\theta = 2\cos^2\theta/2$, $\sin\theta = 2\sin\theta/2\cos\theta/2$.)

(c) Determine the matrix that diagonalizes the matrix A.