Homework No. 09 (Spring 2016)

PHYS 530A: Quantum Mechanics II

Due date: Thursday, 2016 Apr 28, 4.30pm

1. (**20 points.**) A composite system is built out of two angular momenta $j_1 = 7, j_2 = \frac{3}{2}$. Determine the total number of angular momentum states for the composite system.

2. (**20 points.**) We constructed the total angular momentum states of two spin-$\frac{1}{2}$ systems, $j_1 = \frac{1}{2}$, $j_2 = \frac{1}{2}$, by beginning with the total angular momentum state

$$|1,1\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle_{①} \left|\frac{1}{2}, \frac{1}{2}\right\rangle_{②} \tag{1}$$

and using the lowering operator to construct the $|1,0\rangle$ and $|1,-1\rangle$ states. The state $|0,0\rangle$ was then constructed (to within a phase factor) as the state orthogonal to $|1,0\rangle$.

   (a) Repeat this exercise by beginning with the total angular momentum state $|1,-1\rangle$ and using the raising operator to construct $|1,0\rangle$ and $|1,1\rangle$ states.

   (b) Investigate the property of the total angular momentum states under the interchange ①↔②. In particular, find out if each of the total angular momentum states are symmetrical (do not change sign) or antisymmetrical (change sign).

3. (**40 points.**) Let us construct the total angular momentum states for the composite system built out of two angular momenta $j_1 = 2, j_2 = \frac{1}{2}$.

   (a) Determine the total number of states by counting the individual states,

$$\left(\sum_{m_1=-j_1}^{j_1}\right)\left(\sum_{m_2=-j_2}^{j_2}\right). \tag{2}$$

   Repeat this by counting the number of total angular momentum states,

$$\sum_{j=|j_1-j_2|}^{j_1+j_2} \sum_{m=-j}^{j}. \tag{3}$$

   (b) Beginning with $|5/2, 5/2\rangle$ use the lowering operator to build five other states with $j = 5/2$.

   (c) Construct $|3/2, 3/2\rangle$ state by requiring it to be orthogonal to $|5/2, 3/2\rangle$, and be normalized.

Particle	$\vert T, T_3\rangle$	Q
proton	$\vert\frac{1}{2}, \frac{1}{2}\rangle$	1
neutron	$\vert\frac{1}{2}, -\frac{1}{2}\rangle$	0
$\boldsymbol{\pi}^+$	$\vert 1, 1\rangle$	1
$\boldsymbol{\pi}^0$	$\vert 1, 0\rangle$	0
$\boldsymbol{\pi}^-$	$\vert 1, -1\rangle$	-1

Table 1: Isospin assignments for particles.

(d) Beginning with $|3/2, 3/2\rangle$ use the lowering operator to build three other states with $j = 3/2$.

4. (**20 points.**) Construct the total angular momentum state $|3, 3\rangle$ for the composite system built out of two angular momenta $j_1 = 3, j_2 = 1$.

5. (**20 points.**) (Schwinger's QM book, Prob. 3-4a.) Iso(topic) spin T: The nucleon is a particle of isospin $T = \frac{1}{2}$; the state with $T_3 = \frac{1}{2}$ is the proton (p), the state with $T_3 = -\frac{1}{2}$ is the neutron (n). Electric charge of a nucleon is given by $Q = \frac{1}{2} + T_3$. The $\boldsymbol{\pi}$ meson, or pion, has isospin $T = 1$, and electric charge $Q = T_3$, so there are three kinds of pions with different electric charge: $T_3 = 1$ ($\boldsymbol{\pi}^+$), $T_3 = 0$ ($\boldsymbol{\pi}^0$), $T_3 = -1$ ($\boldsymbol{\pi}^-$). (Refer Table 1.)

   Consider the system of a nucleon and a pion. The electric charge of this system is $Q = \frac{1}{2} + T_3$. Check that a system of charge 2, $T_3 = \frac{3}{2}$, is p+$\boldsymbol{\pi}^+$, according to the isospin assignments. Now, if the system is in the state $T = \frac{3}{2}$, $T_3 = \frac{1}{2}$, what is the probability of finding a proton? What is the accompanying $\boldsymbol{\pi}$-meson?