PHYS - 205A (Final Exam) 2016 Fall

Solutions

Prob. 1

$$x(t) = 24t^2 - 3.0t^4$$

$$v(t) = \frac{dx}{dt} = 48t - 12t^3$$

stops $\Rightarrow$ $v(t) = 0$ $\Rightarrow$ $48t - 12t^3 = 0$

$\Rightarrow$ $t(4 - t^2) = 0$

$\Rightarrow$ $t = 0, \pm 2$ s

Thus it stops at $t = 2.0$ s.

$$a(t) = \frac{dv}{dt} = 48 - 36t^2$$

$$a(2) = 48 - 36(2)^2 = -96 \frac{m}{s^2}$$

Prob. 2

$\Delta x =$

$v_{ix} = 3.50 \frac{m}{s}$

$\Delta t =$

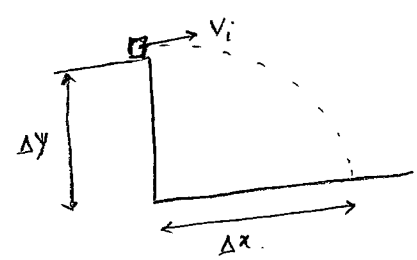
$\Delta y = -1.30$ m

$\Delta t =$

$v_{iy} = 0$

$v_{iy} =$

$a_y = -9.8 \frac{m}{s^2}$.



$$\Delta y = \cancel{v_{iy}}^{=0} \Delta t + \frac{1}{2} a_y \Delta t^2$$

$$-1.30 = \frac{1}{2}(-9.8)\,\Delta t^2$$

$$\Delta t = 0.515 \text{ s}.$$

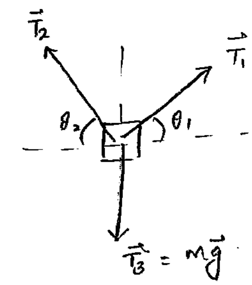
$$\Delta x = v_{ix} \Delta t$$

$$= 3.50 \times 0.515$$

$$= 1.80 \text{ m}$$

Prob. 3

$$\vec{T}_1 + \vec{T}_2 + \vec{T}_3 = m\underbrace{\vec{a}}_{=0}$$



$\underline{x}$

$$T_1 \cos\theta_1 - T_2 \cos\theta_2 = 0$$

$\underline{y}$

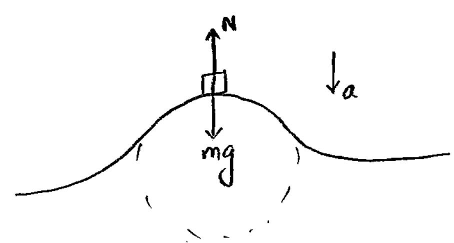
$$T_1 \sin\theta_1 + T_2 \sin\theta_2 - mg = 0$$

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 = mg$$

$$T_1 = \frac{mg\cos\theta_1}{\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2} = \frac{mg\cos\theta_1}{\sin(\theta_1+\theta_2)} = \frac{450\cos 20}{\sin(20+70)} = 423\ \text{N}$$

$$T_2 = \frac{mg\cos\theta_2}{\sin(\theta_1+\theta_2)} = \frac{450\cos 70}{\sin(20+70)} = 154\ \text{N}$$

Prob. 4



$$-mg + N = -\frac{mv^2}{R}$$

threshold is decided by $N=0$.

$$N=0 \Rightarrow \cancel{m}g = \frac{\cancel{m}v_{max}^2}{R}$$

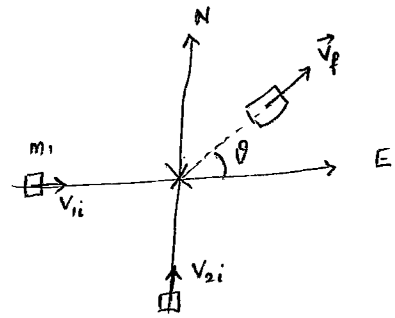
$$v_{max} = \sqrt{gR} = \sqrt{9.8 \times 325} = 56.4\ \frac{m}{s}$$

Prob. 5

(a) Normal force is always perpendicular to direction of motion. Thus, work done by normal force is zero.

(b)
$$W_g = -mg\,\Delta h$$
$$= -4.50 \times 9.8 \times (4.0 - 7.0)$$
$$= 132.3\ \text{J}$$

Prob. 6



$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = (m_1 + m_2)\, \vec{V}_f$$

$$\vec{V}_f = \left(\frac{m_1}{m_1+m_2}\right) \vec{V}_{1i} + \left(\frac{m_2}{m_1+m_2}\right) \vec{V}_{2i}$$

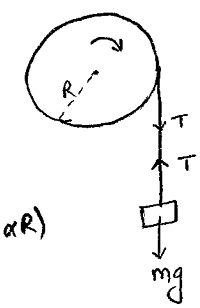
$$= \frac{1}{4} \times 10.0\, \hat{i} + \frac{3}{4} \times 20.0\, \hat{j}$$

$$= \left(2.50\, \hat{i} + 15.0\, \hat{j}\right) \frac{m}{s}$$

magnitude: $|\vec{V}_f| = \sqrt{2.50^2 + 15.0^2} = 15.2 \frac{m}{s}$

direction: $\theta = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right) = \tan^{-1}\left(\frac{15.0}{2.50}\right) = 80.5^\circ$ North of East

Prob. 7



Newton's law (linear)

$$mg - T = ma$$

Newton's law (rotation)

$$TR = I\alpha$$

$$T = \frac{1}{2} MR^2 \frac{\alpha}{R} \qquad (a = \alpha R)$$

$$= \frac{1}{2} M a$$

$$mg - \left(\frac{1}{2} Ma\right) = ma$$

$$mg = \left(m + \frac{M}{2}\right) a$$

$$a = \frac{m}{\left(m + \frac{M}{2}\right)} g = \frac{5.0}{\left(5.0 + \frac{3.00}{2}\right)} \times 9.8 = 7.54 \frac{m}{s^2}$$

Prob. 8

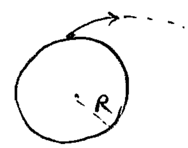
$$I_{disc}\,\omega_i + I_{student}\,\omega_i = I_{disc}\,\omega_f + \underbrace{\cancel{I_{student}}}_{=0}\,\omega_f$$

$$\omega_f = \omega_i\left(1 + \frac{I_{student}}{I_{disc}}\right)$$

$$= \omega_i\left(1 + \frac{mR^2}{I_{disc}}\right)$$

$$= 2.4\ \frac{rad}{s}\left(1 + \frac{150\,kg\,(2.0\,m)^2}{300\,kg\,m^2}\right) = 7.2\ \frac{rad}{s}$$

Prob. 9



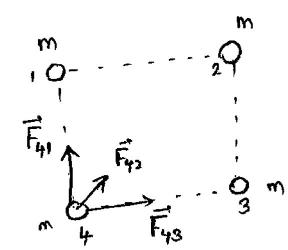
$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 + 0$$

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67\times10^{-11} \times 1.898\times10^{27}\,kg}{69.9\times10^{6}\,m}}$$

$$= 60.2\ \frac{km}{s}$$

Prob. 10



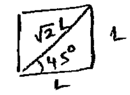
$$\vec{F}_{41} = \hat{i}\,\frac{Gm^2}{L^2} + \hat{j}\,0$$

$$\vec{F}_{42} = \hat{i}\,\frac{Gm^2}{(\sqrt{2}L)^2}\cos 45 + \hat{j}\,\frac{Gm^2}{(\sqrt{2}L)^2}\sin 45$$

$$\vec{F}_{43} = \hat{i}\,0 + \frac{Gm^2}{L^2}\,\hat{j}$$

$$\vec{F}_{tot} = \frac{Gm^2}{L^2}\left(1 + \frac{1}{2\sqrt{2}}\right)\hat{i} + \frac{Gm^2}{L^2}\left(1 + \frac{1}{2\sqrt{2}}\right)\hat{j}$$

$\cos 45 = \frac{1}{\sqrt{2}}$



magnitude: $$|\vec{F}_{tot}| = \frac{Gm^2}{L^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

direction: 45° counter clockwise w.r.t. one mass.