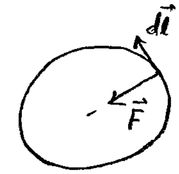
Solutions

PHYS-205A Fall 2016 Midterm Exam 3

Prob 1

(a) At every instant, in uniform circular motion, the direction of the force is perpendicular to the displacement of the particle. Thus,



$$W_{net} = \int \vec{F}_{net} \cdot d\vec{l} = 0$$

(b) $\Delta K = W_{net} = 0$.

Prob. 2

$W = \vec{F} \cdot \vec{d}$

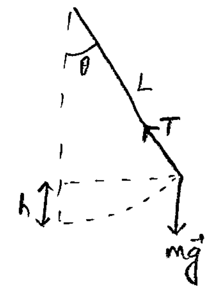
$= F d \cos\theta$ — angle between $\vec{F}$ and $\vec{d}$.

(a)
$$W_{O\to A\to C} = W_{O\to A} + W_{A\to C}$$
$$= mg\,x \underbrace{\cos 90}_{=0} + mg\,y \cos(180)$$
$$= -mgy = -5.10 \times 9.8 \times 4.50 = -221\ \text{J}$$

(b) Work done by the gravitational force is independent of the path taken by the particle. So,

$$W_{O\to C} = W_{O\to A\to C} = -221\ \text{J}$$

Prob. 3



(a) Force of tension is always perpendicular to the displacement $d\vec{l}$. Thus,

$$W_T = \int \vec{T} \cdot d\vec{l} = 0$$

(b)

$$\Delta K = mgh + \underbrace{W_T}_{=0}$$

$$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g h$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.134}$$

$$= 1.62 \ \frac{m}{s}$$

$$h = L - L\cos\theta$$
$$= 1.0 - 1.0 \cos 30$$
$$= 0.134 \text{ m}$$

Prob. 4

(a)

$$K^A + U_g^A = K^B + U_g^B$$

$$\frac{1}{2} \cancel{m} \underbrace{v_A^2}_{=0} + \cancel{m} g h_A = \frac{1}{2} \cancel{m} V_B^2 + \cancel{m} g \underbrace{h_B}_{=0}$$

$$V_B = \sqrt{2 g h_A} = \sqrt{2 \times 9.8 \times 2.0} = 6.26 \ \frac{m}{s}$$

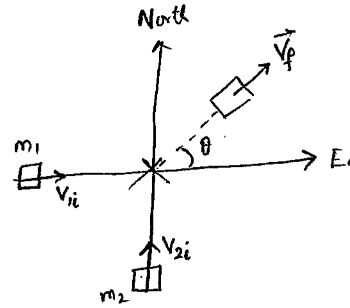
(b)

$$K^B + \underbrace{U_g^B}_{=0} + U_s^B = K^c + \underbrace{U_g^c}_{=0} + U_s^c$$

$$\frac{1}{2} m V_B^2 + \frac{1}{2} k \underbrace{x_B^2}_{=0} = \frac{1}{2} m \underbrace{V_c^2}_{=0} + \frac{1}{2} k x_c^2$$

$$x_c = \sqrt{\frac{m}{k}} \ V_B$$

$$= \sqrt{\frac{30.0}{5.0 \times 10^4}} \ 6.26 = 14.0 \text{ cm}$$



Prob. 5

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = (m_1 + m_2)\vec{V}_f$$

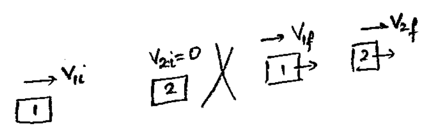
$$\vec{V}_f = \frac{m_1}{m_1+m_2} V_{1i}\,\hat{i} + \frac{m_2}{m_1+m_2} V_{2i}\,\hat{j}$$

$$= \frac{2000}{2000+6000} \times 20.0\,\hat{i} + \frac{6000}{2000+6000} \times 10.0\,\hat{j}$$

$$= 5.00\,\hat{i} + 7.50\,\hat{j}$$

magnitude: $|\vec{V}_f| = \sqrt{5.00^2 + 7.50^2} = 9.01\ \frac{m}{s}$

direction: $\theta = \tan^{-1}\left(\frac{7.5}{5.00}\right) = 56.3^\circ$ (North of East)



Prob. 6

Using Eqs. (21) in equation sheet

$$V_{1f} = \left(\frac{m_1-m_2}{m_1+m_2}\right) V_{1i} + \left(\frac{2m_2}{m_1+m_2}\right) \underbrace{V_{2i}}_{=0}$$

$$= \left(\frac{m_1-m_2}{m_1+m_2}\right) V_{1i} = \left(\frac{1.00 - 10.0}{1.00 + 10.0}\right) \times 10.0\ \frac{m}{s} = -8.18\ \frac{m}{s}$$

$$V_{2f} = \left(\frac{2m_1}{m_1+m_2}\right) V_{1i} + \left(\frac{m_2-m_1}{m_1+m_2}\right) \underbrace{V_{2i}}_{=0}$$

$$= \left(\frac{2m_1}{m_1+m_2}\right) V_{1i} = \left(\frac{2 \times 1.00}{1.00 + 10.0}\right) \times 10.0\ \frac{m}{s} = +1.82\ \frac{m}{s}$$

Prob. 7

(a) $$\vec{F} = -\hat{i}\frac{\partial U}{\partial x} - \hat{j}\frac{\partial U}{\partial y} - \hat{k}\frac{\partial U}{\partial z}$$

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\frac{a}{\sqrt{x^2+y^2+z^2}} = -\frac{1}{2}\frac{a\ 2x}{(x^2+y^2+z^2)^{3/2}} = -\frac{ax}{r^3}$$

$$\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\frac{a}{\sqrt{x^2+y^2+z^2}} = -\frac{1}{2}\frac{a\ 2y}{(x^2+y^2+z^2)^{3/2}} = -\frac{ay}{r^3}$$

$$\frac{\partial U}{\partial z} = \frac{\partial}{\partial z}\frac{a}{\sqrt{x^2+y^2+z^2}} = -\frac{1}{2}\frac{a\ 2z}{(x^2+y^2+z^2)^{3/2}} = -\frac{az}{r^3}$$

$$\vec{F} = +\frac{a(x\hat{i}+y\hat{j}+z\hat{k})}{r^3} = \frac{a\vec{r}}{r^3}$$

(b) The direction of the force is given by $\vec{r}$, which is directed away from the origin. Thus, the force is repulsive.

Problem 8

(a) $$\frac{dm}{dx} = a x^2$$

$$\int dm = \int_0^L a x^2 dx$$

$$M = a\frac{L^3}{3} = 3.00\ \frac{kg}{m^3} \times \frac{(5.00\ m)^3}{3} = 125\ kg$$

(b) $$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^L x\, \cancel{a} x^2 dx}{\int_0^L \cancel{a} x^2 dx}$$

$$= \frac{\int_0^L x^3 dx}{\int_0^L x^2 dx} = \frac{\frac{L^4}{4} - 0}{\frac{L^3}{3} - 0} = \frac{3}{4}L$$

$$= 3.75\ m$$