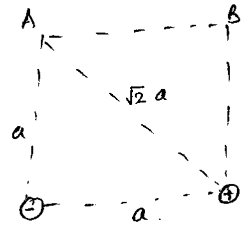
PHYS-205B, FALL 2016, Midterm Exam 2 — Solutions

Prob. 1



(a) $V_A = -\dfrac{kq}{a} + \dfrac{kq}{\sqrt{2}\,a} = -\dfrac{kq}{a}\left(1 - \dfrac{1}{\sqrt{2}}\right)$

$= -\dfrac{8.99\times10^{9} \times 5.00\times10^{-9}\,C}{5.00\times10^{-2}\,m}\left(1 - \dfrac{1}{\sqrt{2}}\right) = -263\ V$

(b) $V_B = -\dfrac{kq}{\sqrt{2}\,a} + \dfrac{kq}{a} = +\dfrac{kq}{a}\left(1 - \dfrac{1}{\sqrt{2}}\right) = +263\ V$

(c) $V_B - V_A = +263 - (-263) = +527\ V$

(d) $\Delta U = Q\,\Delta V$

$= Q\,(V_B - V_A) = 2.00\times10^{-6}\,C \times 527\ V$

$= 1.05\ mJ$

Prob. 2

$V = a + bx^2$

$\vec{E} = -\hat{i}\dfrac{\partial V}{\partial x} - \hat{j}\dfrac{\partial V}{\partial y} - \hat{k}\dfrac{\partial V}{\partial z}$

$\dfrac{\partial V}{\partial x} = 2bx$

$\dfrac{\partial V}{\partial y} = 0$

$\dfrac{\partial V}{\partial z} = 0$

$= -\hat{i}\,2bx - \hat{j}\,0 - \hat{k}\,0$

$= -\hat{i}\,2\left(-450\ \dfrac{V}{cm^2}\right) 2.00\ cm$

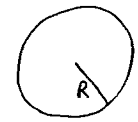
$= -\hat{i}\ 1800\ \dfrac{V}{cm}$

Magnitude: $1800\ \dfrac{V}{cm}$

Direction: along negative $\hat{x}$.

Prob. 3

(a) $r = 8.00\,cm < R = 14.0\,cm$



$R = 14.0\ cm$

Potential inside a conductor is equal to the potential on the surface of a conductor.

$$V = \frac{kQ}{R} = \frac{8.99\times10^{+9} \times 8.0\times10^{-6}}{14.0\times10^{-2}} = 5.14\times10^{5}\ V$$

(b) $r = 16.0\ cm > R = 14.0\,cm$

$$V = \frac{kQ}{r} = \frac{8.99\times10^{9} \times 8.0\times10^{-6}}{16.0\times10^{-2}} = 4.50\times10^{5}\ V$$

Prob. 4

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$\text{Cost} = \left(0.12\ \frac{USD}{kWh}\right) \times (\text{Energy consumed in 2 weeks})$$

$$= \left(0.12\ \frac{USD}{10^3\ \cancel{Wh}}\right) \times \left(100.0\ \cancel{W} \times 14 \times 24\ \cancel{h}\right)$$

$$= 0.12 \times \frac{1}{10^3} \times 100.0 \times 14 \times 24\ USD$$

$$= 4.03\ USD$$

Prob. 5

$$R_1 = R_2 = \cdots = R_{100} = R = 200.0\,\Omega$$

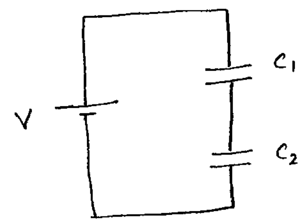
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{100}}$$

$$= \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}$$

$$\frac{1}{R_{eq}} = \frac{100}{R}$$

$$\Rightarrow R_{eq} = \frac{R}{100} = \frac{200.0\,\Omega}{100} = 2.00\,\Omega.$$

Prob. 6.



(a) $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.00} + \frac{1}{2.00} = \frac{3}{2.00}$

$\Rightarrow \quad C_{eq} = 0.667\ \mu F$

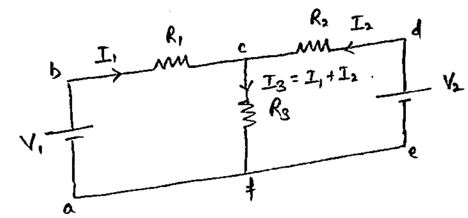
(b) $Q_1 = Q_2 = Q = V C_{eq}$

$= 10.0 \times 0.667\,\mu = 6.67\,\mu C$

(c) $V_1 = \frac{Q_1}{C_1} = \frac{6.67\,\mu C}{1.00\,\mu F} = 6.67\ V$

$V_2 = \frac{Q_2}{C_2} = \frac{6.67\,\mu C}{2.00\,\mu F} = 3.33\ V$

Prob. 7



loop abcfa :

$+V_1 - I_1 R_1 - I_3 R_3 = 0$

$+10 - I_1\,100 - (I_1 + I_2)\,300 = 0$

$40 I_1 + 30 I_2 = 1$

loop edcfe :

$+V_2 - I_2 R_2 - I_3 R_3 = 0$

$+20 - I_2\,200 - (I_1 + I_2)\,300 = 0$

$30 I_1 + 50 I_2 = 2$

$40 I_1 + 30 I_2 = 1$

$30 I_1 + 50 I_2 = 2$

$I_1 = \frac{1 \times 50 - 2 \times 30}{40 \times 50 - 30 \times 30} = -9.09\ mA$

$I_2 = \frac{40 \times 2 - 30 \times 1}{40 \times 50 - 30 \times 30} = +45.5\ mA$

$I_3 = I_1 + I_2$

$= -9.09 + 45.5 = +36.4\ mA$

Direction: from $c \rightarrow f$.

Prob. 8

(a) $+V - IR - \frac{Q}{C} = 0$

(b) Maximum charge $= Q(\infty) = CV$

$$Q(t) = CV\left[1 - e^{-\frac{t}{RC}}\right]$$

$$0.70\, \cancel{CV} = \cancel{CV}\left[1 - e^{-\frac{t}{RC}}\right]$$

$$0.70 = 1 - e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = 1 - 0.70 = 0.30$$

$$\ln e^{-\frac{t}{RC}} = \ln 0.30$$

$$-\frac{t}{RC} = \ln 0.30$$

$$t = -RC \ln 0.30$$

$$= -30.0 \times 10^{3}\ \Omega \times 10.0 \times 10^{-6}\ F\ \ln(0.30)$$

$$= 0.361 \text{ seconds.}$$