Midterm Exam No. 01 (Fall 2016)

PHYS 530B: Quantum Mechanics II

Date: 2016 Oct 3

1. (**40 points.**) A mass m oscillates about an equilibrium point with angular frequency ω. This motion, the harmonic oscillator, is described by the Hamiltonian

$$H_0(x,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, \tag{1}$$

in conjunction with the Heisenberg relation $[x,p] = i\hbar$. Let us chose our units such that m=1 and $\omega = 1$. The above Hamiltonian takes the form

$$H_0 = \left(y^\dagger y + \frac{1}{2} \right) \hbar \tag{2}$$

in terms of the operators

$$x + ip = \sqrt{2\hbar}\, y, \tag{3a}$$

$$x - ip = \sqrt{2\hbar}\, y^\dagger. \tag{3b}$$

The harmonic oscillator is characterized by the algebra

$$[y, y^\dagger] = 1. \tag{4}$$

The eigen basis set $|n'\rangle$ of the harmonic oscillator satisfies the eigenvalue equation

$$y^\dagger y |n'\rangle = n'|n'\rangle, \qquad n' = 0, 1, 2, \ldots. \tag{5}$$

Let us place a charge q on the mass m, such that it interacts with a weak electric field $\mathbf{E}$. We choose the direction of the electric field $\mathbf{E}$ to be in the direction of the oscillations of the mass. The Hamiltonian of the oscillator in the presence of the electric field is described by

$$H = H_0 - qEx. \tag{6}$$

In the presence of the electric field the oscillator ceases to stay in the stationary state $|n'\rangle$, and makes transitions to another state. These transitions are described by the matrix elements

$$\langle n''|H|n'\rangle. \tag{7}$$

Find a selection rule that states which elements are not zero.
Hint: Use equations of motion

$$\frac{dF}{dt} = \frac{1}{i\hbar}[F, H_0]. \tag{8}$$

2. (**20 points.**) In Problem 1 of Homework No. 03, evaluate the inverse Fourier transformation

$$\psi_{100}(\mathbf{r}) = (2\pi\hbar)^{-\frac{3}{2}} \int d^3\mathbf{p}\, e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}}\, \psi_{100}(\mathbf{p}) \tag{9}$$

   and verify that this is indeed equal to the result obtained for $\psi_{100}(\mathbf{r})$ in Item (a) of the problem.
   Hint: Use the integral

$$\int_0^\infty \frac{x\, dx}{(x^2+1)^2} \sin ax = \frac{\pi a}{4} e^{-a}. \tag{10}$$

3. (**20 points.**) Submit Problem 2 of Homework No. 03.

4. (**20 points.**) Write a summary of the discussion in the following article:
   'Rydberg Atoms in "Circular" States,' by R. G. Hulet and D. Kleppner,
   Phys. Rev. Lett. 51 (1983) 1430.