# Midterm Exam No. 02 (Fall 2017)

## PHYS 440: Quantum Mechanics

Date: 2017 Oct 17

1. (**20 points.**) A homogeneous magnetic field $\mathbf{B}$ is characterized by the vector potential
$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}. \tag{1}$$
   (a) Evaluate $\boldsymbol{\nabla} \times \mathbf{A}$. (Hint: $\boldsymbol{\nabla} \times \mathbf{A} = \mathbf{B}$.)
   (b) Now, for the case of $\mathbf{B} = (0, 0, B)$, pointing in the $z$ direction, show that $\mathbf{A} = (0, Bx, 0)$ is a solution. Show that $\mathbf{A} = (-By, 0, 0)$ is also a solution. Show that $\mathbf{A} = (-By/2, Bx/2, 0)$ is also a solution. Find another solution.

2. (**20 points.**) The components of $\boldsymbol{\sigma}$ are Pauli matrices and satisfy the commutation relations (of spin)
$$\big[\sigma_i, \sigma_j\big] = 2i\varepsilon_{ijk}\sigma_k. \tag{2}$$
The components of $\boldsymbol{\tau}$ are also Pauli matrices and satisfy the commutation relations (of isospin)
$$\big[\tau_i, \tau_j\big] = 2i\varepsilon_{ijk}\tau_k. \tag{3}$$
The operators $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ act on disjoint Hilbert spaces. Find the eigenvalues of the operator construction
$$H = \sigma_x + \tau_y, \tag{4}$$
which is a short hand for the expression
$$H = \sigma_x \otimes 1 + 1 \otimes \tau_y. \tag{5}$$

3. (**20 points.**) The ground eigenstate of a harmonic oscillator satisfies the equation
$$y|0\rangle = 0, \tag{6}$$
where
$$y = \frac{1}{\sqrt{2}}\left(x + ip\right). \tag{7}$$
Construct the differential equation satisfied by the ground eigenstate in the momentum basis,
$$\psi_0(p') = \langle p'|0\rangle. \tag{8}$$
Solve the differential equation and find the normalized ground eigenstate.

4. (**20 points.**) A charge in a uniform magnetic field, in the absence of an electric field, is described by the Hamiltonian

$$H(\mathbf{x}, \mathbf{p}) = \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2, \tag{9}$$

where

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}. \tag{10}$$

Evaluate the commutation relations in the Hamilton equations of motion

$$\frac{d\mathbf{x}}{dt} = \frac{1}{i\hbar}\big[\mathbf{x}, H\big], \tag{11}$$

$$\frac{d\mathbf{p}}{dt} = \frac{1}{i\hbar}\big[\mathbf{p}, H\big], \tag{12}$$

to obtain equations of motion for the charge. Are these consistent with the Lorentz force? Justify.