Midterm Exam No. 03 (Fall 2017)

PHYS 440: Quantum Mechanics

Date: 2017 Nov 16

1. (**20 points.**) A particle of mass m and charge q moving in a uniform magnetic field $\mathbf{B}$ experiences a velocity dependent force $\mathbf{F}$ given by the expression

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}, \tag{1}$$

where $\mathbf{v}(t) = d\mathbf{x}/dt$ is the velocity of the particle in terms of its position $\mathbf{x}(t)$. Choose the magnetic field to be along the positive z direction, $\mathbf{B} = B\hat{\mathbf{z}}$.

(a) For the case when the particle starts at rest at the origin at time $t = 0$, use the initial conditions

$$\mathbf{v}(0) = -\frac{v_0}{\sqrt{2}}\,\hat{\mathbf{x}} + \frac{v_0}{\sqrt{2}}\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{z}}, \tag{2}$$

$$\mathbf{x}(0) = 0\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{z}}, \tag{3}$$

to solve the differential equation in Eq. (1) to find the position $\mathbf{x}(t)$ and velocity $\mathbf{v}(t)$ as a function of time. Use $\omega = qB/m$.
Hint: Second-order differential equations for v_x and v_y introduce two arbitrary constants each. Integrations of these equations, to construct the respective positions, leads to another two arbitrary constants. Thus, there are a total of six arbitrary constants. Four of these are determined by the initial conditions. The remaining two are determined by the coupling in the components of the velocity, given by the first-order differential equations in Eq. (1).

(b) In particular, prove that the particle takes a circular path. What is the radius of the circle? Determine the coordinates of the center of the circle?

2. (**20 points.**) Consider the unperturbed Hamiltonian to be

$$\gamma = J_x, \tag{4}$$

where $\mathbf{J}$ is the angular momentum. Let $j = 1/2$. Consider the perturbation

$$\gamma + \delta\gamma = J_x + \alpha J_z. \tag{5}$$

Determine the eigenvalues and eigenfunctions of $\gamma + \delta\gamma$ to the leading order in α. Further, verify that the eigenfunctions are orthonormal upto the leading order in α.

3. (**20 points.**) Let us construct the total angular momentum states for the composite system built out of two angular momenta $j_1 = 1, j_2 = \frac{1}{2}$. Beginning with $|3/2, 3/2\rangle$ use the lowering operator to build three other states with $j = 3/2$.

4. (**20 points.**) Consider the construction

$$K(\lambda) = e^{-\lambda A} B e^{\lambda A} \tag{6}$$

in terms of two operators A and B. Show that

$$\frac{\partial K}{\partial \lambda} = [K, A]. \tag{7}$$

Evaluate the expression for the higher derivative

$$\frac{\partial^2 K}{\partial \lambda^2}. \tag{8}$$