## Homework No. 08 (Spring 2018) PHYS 530A: Quantum Mechanics II

Due date: Tuesday, 2018 Mar 27, 4.30pm

1. (40 points.) Consider the operator constructions

$$\sigma_{+} = \frac{1}{2}(\sigma_x + i\sigma_y) \quad \text{and} \quad \sigma_{-} = \frac{1}{2}(\sigma_x - i\sigma_y).$$
 (1)

(a) Show that

$$\sigma_{+}|-\rangle = |+\rangle, \qquad \qquad \sigma_{+}|+\rangle = 0, \qquad (2a)$$

$$|-\rangle = 0,$$
  $\sigma_{-}|+\rangle = |-\rangle.$  (2b)

- (b) Evaluate  $\sigma_+^2$ ,  $\sigma_-^2$ ,  $\sigma_+\sigma_-$ ,  $\sigma_-\sigma_+$ , the commutation relation  $[\sigma_+, \sigma_-]$ , and the anticommutation relation  $\{\sigma_+, \sigma_-\}$ .
- 2. (40 points.) Consider the operator

$$\gamma = y\sigma_+ + y^{\dagger}\sigma_- + \epsilon\sigma_z, \tag{3}$$

where  $\sigma_+ = (\sigma_x + i\sigma_y)/2$  and  $\sigma_- = (\sigma_x - i\sigma_y)/2$ .

 $\sigma$ 

(a) Let  $|n', \pm 1\rangle$ , n' = 0, 1, 2, ..., be the common eigenvectors of  $y^{\dagger}y$  and  $\sigma_z$ . Consider the subspace spanned by the eigenvectors  $|n', +1\rangle$  and  $|n' + 1, -1\rangle$ . Construct the matrix representation of the operator  $\gamma$  in this subspace by evaluating the elements in

$$\gamma = \begin{bmatrix} \langle n', +1|\gamma|n', +1 \rangle & \langle n', +1|\gamma|n'+1, -1 \rangle \\ \langle n'+1, -1|\gamma|n', +1 \rangle & \langle n'+1, -1|\gamma|n'+1, -1 \rangle \end{bmatrix}.$$
(4)

- (b) Find the eigenvalues of the operator  $\gamma$ .
- (c) Find the two normalized eigenvectors (up to a phase) of the operator  $\gamma$  in the subspace discussed above, and express them in terms of the respective common eigenvectors. In particular, show that

$$|n' + \frac{1}{2}, +1\rangle = \cos\frac{\alpha_n}{2}|n', +1\rangle + \sin\frac{\alpha_n}{2}|n' + 1, -1\rangle,$$
(5a)

$$|n' + \frac{1}{2}, -1\rangle = -\sin\frac{\alpha_n}{2}|n', +1\rangle + \cos\frac{\alpha_n}{2}|n' + 1, -1\rangle,$$
 (5b)

where

$$\tan \alpha_n = \frac{\sqrt{n'+1}}{\epsilon}.$$
(6)