## Homework No. 10 (Spring 2018)

## PHYS 530A: Quantum Mechanics II

Due date: Tuesday, 2018 Apr 17, 4.30pm

- 1. (20 points.) A composite system is built out of two angular momenta  $j_1 = 7, j_2 = \frac{3}{2}$ . Determine the total number of angular momentum states for the composite system.
- 2. (20 points.) We constructed the total angular momentum states of two spin- $\frac{1}{2}$  systems,  $j_1 = \frac{1}{2}, j_2 = \frac{1}{2}$ , by beginning with the total angular momentum state

$$|1,1\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle_{\textcircled{0}} \left|\frac{1}{2},\frac{1}{2}\right\rangle_{\textcircled{0}}$$
(1)

and using the lowering operator to construct the  $|1,0\rangle$  and  $|1,-1\rangle$  states. The state  $|0,0\rangle$  was then constructed (to within a phase factor) as the state orthogonal to  $|1,0\rangle$ .

- (a) Repeat this exercise by beginning with the total angular momentum state  $|1, -1\rangle$  and using the raising operator to construct  $|1, 0\rangle$  and  $|1, 1\rangle$  states.
- (b) Investigate the property of the total angular momentum states under the interchange ①↔②. In particular, find out if each of the total angular momentum states are symmetrical (do not change sign) or antisymmetrical (change sign).
- 3. (40 points.) Let us construct the total angular momentum states for the composite system built out of two angular momenta  $j_1 = 2, j_2 = \frac{1}{2}$ .
  - (a) Determine the total number of states by counting the individual states,

$$\left(\sum_{m_1=-j_1}^{j_1}\right)\left(\sum_{m_2=-j_2}^{j_2}\right).$$
(2)

Repeat this by counting the number of total angular momentum states,

$$\sum_{j=|j_1-j_2|}^{j_1+j_2} \sum_{m=-j}^{j}$$
 (3)

- (b) Beginning with  $|5/2, 5/2\rangle$  use the lowering operator to build five other states with j = 5/2.
- (c) Construct  $|3/2, 3/2\rangle$  state by requiring it to be orthogonal to  $|5/2, 3/2\rangle$ , and be normalized.
- (d) Beginning with  $|3/2, 3/2\rangle$  use the lowering operator to build three other states with j = 3/2.