Homework No. 12 (Spring 2018)

PHYS 530A: Quantum Mechanics II

Due date: Thursday, 2018 Apr 26, 4.30pm

1. (**20 points.**) The polar equation of a conic of eccentricity ε is

$$r = \frac{a(1-\varepsilon^2)}{1-\varepsilon\cos\theta}, \tag{1}$$

where $2a$ is the major-axis of the ellipse. The directrix of an ellipse is a line perpendicular to the major-axis at a distance d from the focus (origin). For an ellipse, the ratio between the radial distance of a point on the ellipse from the origin, and the distance of the point from the directrix, is the eccentricity. Thus, determine d in terms of a and ε.

2. (**20 points.**) The components of the position and momentum operator, $\mathbf{r}$ and $\mathbf{p}$, respectively, satisfy the commutation relations $[r_i, p_j] = i\hbar\delta_{ij}$. Show that

$$r^2\,\mathbf{p} - \mathbf{p}\,r^2 = 2i\hbar\mathbf{r}. \tag{2}$$

3. (**20 points.**) Evaluate the commutation relation between $1/r$, the inverse of the magnitude of the position operator $\mathbf{r}$, and the angular momentum operator $\mathbf{L}$,

$$\left[\frac{1}{r}, \mathbf{L}\right], \tag{3}$$

which is encountered, for example, in the analysis of hydrogen atom.

4. (**20 points.**) Using commutation relations between position $\mathbf{r}$, linear momentum $\mathbf{p}$, and angular momentum $\mathbf{L}$, verify the relation

$$\mathbf{p}\times\mathbf{L}\cdot\mathbf{p} = c\,i\hbar\,p^2, \tag{4}$$

where c is a number. Determine c.