Homework No. 13 (2018 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2018 Dec 7, 2:00 PM, in class

1. (**20 points.**) A monochromatic plana wave is characterized by the direction of propagation $\mathbf{k}$, the electric field $\mathbf{E}$, and the magnetic field $\mathbf{B}$, that satisfy

$$\mathbf{k}\cdot\mathbf{E}=0,\quad \mathbf{k}\cdot\mathbf{B}=0,\quad \mathbf{k}\times\mathbf{E}=\omega\mathbf{B},\quad \mathbf{k}\times\mathbf{B}=-\frac{\omega}{c^2}\mathbf{E}. \tag{1}$$

   These equations further imply $k=\omega/c$. At the interface of two materials, Snell's law of refraction states that the direction of propagation $\mathbf{k}$ bends towards the normal vector to the interface when the plane wave goes from a region of lower refractive index to a region of higher refractive index. Verify that the direction of the electric field bends away from the normal vector to the interface in the same scenario.

2. (**20 points.**) Consider a uniformly polarized half-slab, that occupies half of space, and has the direction of its polarization transverse to the direction $\hat{\mathbf{z}}$ normal to the surface of slab, described by

$$\mathbf{P}(\mathbf{r})=\sigma\,\hat{\mathbf{y}}\,\theta(-z), \tag{2}$$

   where σ is the polarization per unit area of the slab. Determine the effective charge density by evaluating

$$\rho_{\text{eff}}(\mathbf{r})=-\boldsymbol{\nabla}\cdot\mathbf{P}. \tag{3}$$

   What can you say about the electric field, inside and outside the slab?