Homework No. 05 (2019 Fall)

PHYS 301: Theoretical Methods in Physics

Due date: Friday, 2019 Sep 27, 10:00 AM, in class

0. Keywords: Index notation; Dyadic notation; Kronecker delta-symbol; Levi-Civita symbol.

1. (**10 points.**) Using index notation and the antisymmetric property of the Levi-Civita symbol show that
$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = -\mathbf{A}\cdot\mathbf{C}\times\mathbf{B}. \tag{1}$$

2. (**20 points.**) In three dimensions the Levi-Civita symbol is given in terms of the determinant of the Kronecker delta,
$$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix} \tag{2a}$$
$$= \delta_{il}(\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}) - \delta_{im}(\delta_{jl}\delta_{kn} - \delta_{jn}\delta_{kl}) + \delta_{in}(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}). \tag{2b}$$
Using the above identity show that
$$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}. \tag{3}$$
Thus, derive the vector identity (using index notation)
$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C}). \tag{4}$$

3. (**20 points.**) Verify the following identities:
$$\boldsymbol{\nabla} r = \frac{\mathbf{r}}{r} = \hat{\mathbf{r}}, \tag{5a}$$
$$\boldsymbol{\nabla}\,\mathbf{r} = \mathbf{1}. \tag{5b}$$
Further, show that
$$\boldsymbol{\nabla}\cdot\mathbf{r} = 3, \tag{6a}$$
$$\boldsymbol{\nabla}\times\mathbf{r} = 0. \tag{6b}$$
Here r is the magnitude of the position vector $\mathbf{r}$, and $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of $\mathbf{r}$.

4. (**20 points.**) (Based on Problem 1.13, Griffiths 4th edition.)
Show that
$$\boldsymbol{\nabla} r^2 = 2\mathbf{r}. \tag{7}$$
Then evaluate $\boldsymbol{\nabla} r^3$. Show that
$$\boldsymbol{\nabla}\frac{1}{r} = -\frac{\hat{\mathbf{r}}}{r^2}. \tag{8}$$
Then evaluate $\boldsymbol{\nabla}(1/r^2)$.