Homework No. 13 (2019 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Wednesday, 2019 Dec 4, 2:00 PM, in class

0. Keywords: Method of images in electrostatics

0. Problems 1, 4, and 5 are to be submitted for assessment. Rest are for practice.

1. (**20 points.**) A grounded perfectly conducting thin plate is placed at $z = 0$ plane. A positive charge q is placed at $\mathbf{r} = d\,\hat{\mathbf{z}}$. Using method of images determine the direction and magnitude of the electric field at the point $\mathbf{r} = d\,\hat{\mathbf{x}} + 2d\,\hat{\mathbf{z}}$.

2. (**20 points.**) A grounded perfectly conducting thin plate is placed at $z = 0$ plane. A positive charge q is placed at $\mathbf{r} = a\,\hat{\mathbf{z}}$. What is the electric field at the point $\mathbf{r} = -a\,\hat{\mathbf{z}}$?

3. (**20 points.**) A grounded perfect electric conductor with a planar surface occupies half of space. Two identical positive charges are placed a distance a in front of the conductor such that the distance between the two charges is $2a$. Determine the magnitude and direction of electric field at the point midway between the two charges.

4. (**20 points.**) A thin grounded perfect conductor occupies the $z = 0$ plane. A point charge q_1 is placed on one side of this conductor and another point charge q_2 is placed on the other side. The line connecting the position of the two charges is not necessarily perpendicular to the conducting plane. Let us ignore forces other than electrostatic forces in this analysis.

   (a) Identify and list the forces acting on charge q_1. Qualitatively determine the total force on charge q_1.

   (b) Identify and list the forces acting on charge q_2. Qualitatively determine the total force on charge q_2.

   (c) Identify and list the forces acting on the conductor. Qualitatively determine the total force on the conductor.

   (d) Does the conductor experience a torque?

5. (**20 points.**) Consider two grounded, thin, perfect conductors occupying half planes extending radially outward from the z axis. Let these planes intersect at the z axis making an angle of 120° between them. That is, say, the two planes are $\theta = \pi/3$ and $\theta = -\pi/3$. Place a point charge on the plane $\theta = \pi/6$ as described in Figure 1. Determine the resulting image charge configuration, assuming that the method of images extends to these configurations analogous to optical images in a mirror.

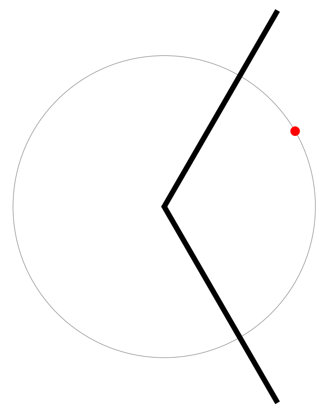

Figure 1: A charge near two intersecting grounded perfect conductors.

6. (**20 points.**) Consider two grounded, thin, perfect conductors occupying half planes extending radially outward from the z axis. Let these planes intersect at the z axis making an angle of 120° between them. That is, say, the two planes are $\theta = \pi/3$ and $\theta = -\pi/3$. Place a point charge on the plane $\theta = \pi/6$ as described in Figure 1. The resulting image charge configuration, assuming that the method of images extends to these configurations analogous to optical images in a mirror was found the previous problem Let us vary the position of the point charge slightly such that it is on the plane $\theta = (\pi/6) + \varepsilon$, where $\varepsilon > 0$. Find the resulting variation in the image charge configuration.

7. (**20 points.**) A point charge q is placed near a perfectly conducting plate.

   (a) Will the charge q experience a force?

   (b) If yes, calculate the force of attraction/repulsion between the charge and conducting plate when the charge is a distance a away from the plate.

   (c) If no, why not?

8. (**Example.**) A grounded perfectly conducting thin plate is placed at $z = 0$ plane. A positive charge q is placed at $\mathbf{r} = a\,\hat{\mathbf{z}}$, $a > 0$. The electric potential for this configuration is given by

$$\phi(x, y, z) = \begin{cases} 0, & z < 0, \\ \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{\sqrt{x^2 + y^2 + (z-a)^2}} - \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{\sqrt{x^2 + y^2 + (z+a)^2}}, & 0 < z. \end{cases} \tag{1}$$

   (a) Show that the electric potential is continuous at the surface of the conductor. That is, for an infinitely small δ,

$$\phi(x, y, 0 - \delta) = \phi(x, y, 0 + \delta). \tag{2}$$

(b) For a fixed $\rho = \sqrt{x^2 + y^2} \neq 0$ and $q > 0$ plot the electric potential as a function of z for $-\infty < z < \infty$. This has been plotted in Fig. 2 Is the force on the charge attractive or repulsive for $z > z_0$ and $z < z_0$? How much energy is required to move a test charge from a distance very far from the conducting plate to the point (ρ, z) on the surface of conductor. How does this plot change for $q < 0$?

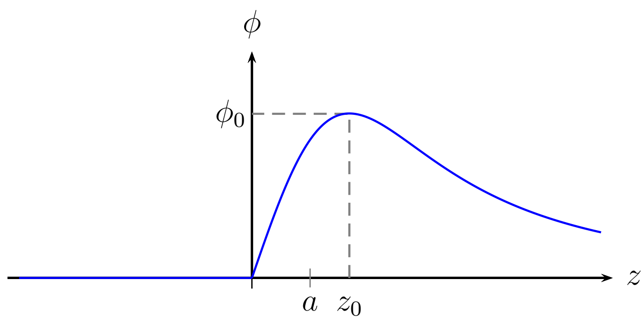

Figure 2: Electric potential as a function of z, for fixed ρ, for a positive charge q placed in front of a conducting plane.