

Homework No. 01 (Fall 2019)

PHYS 520A: Electromagnetic Theory I

Due date: Monday, 2019 Aug 26, 4.00pm

1. (Ref. Schwinger et al., Problem 1, Chapter 1.) Verify the following identities explicitly:

(a) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A},$

(b) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = (\nabla \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\nabla \times \mathbf{B}),$

(c) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) + \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) + \mathbf{C} \times (\mathbf{A} \times \mathbf{B}) = 0,$

(d) $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) - \mathbf{B} \times (\nabla \times \mathbf{A}) - (\mathbf{A} \times \nabla) \times \mathbf{B} + (\mathbf{B} \times \nabla) \times \mathbf{A}.$

2. For the position vector

$$\mathbf{r} = r \hat{\mathbf{r}} = x \hat{\mathbf{i}} + y \hat{\mathbf{j}} + z \hat{\mathbf{k}}, \quad (1)$$

show that

$$\nabla r = \hat{\mathbf{r}}, \quad \nabla \mathbf{r} = \mathbf{1}, \quad \nabla \cdot \mathbf{r} = 3, \quad \text{and} \quad \nabla \times \mathbf{r} = 0. \quad (2)$$

Further, show that for $n \neq 3$

$$\nabla \frac{\mathbf{r}}{r^n} = \mathbf{1} \frac{1}{r^n} - \mathbf{r} \mathbf{r} \frac{n}{r^{n+2}}, \quad \nabla \cdot \frac{\mathbf{r}}{r^n} = \frac{(3-n)}{r^n}, \quad \text{and} \quad \nabla \times \frac{\mathbf{r}}{r^n} = 0. \quad (3)$$

For $n = 3$ use divergence theorem to show that

$$\nabla \cdot \frac{\mathbf{r}}{r^n} = 4\pi \delta^{(3)}(\mathbf{x}). \quad (4)$$