Homework No. 01 (2020 Spring)

PHYS 301: THEORETICAL METHODS IN PHYSICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Monday, 2020 Jan 20, 9:00 AM, in class

0. Problems 1, 2, and 5 are to be submitted for assessment. Rest are for practice.

0. Keywords: Complex numbers, nth roots of unity.

0. Reference: Chapters VIII, IX, and XI, in Advanced Trigonometry, by Durell and Robson, which is a popular high school textbook.

1. (**10 points.**) Find the real and imaginary part of the following functions of the complex variable $z = x + iy$.

$$f = \frac{1}{z^2}, \tag{1a}$$

$$f = e^{iz}, \tag{1b}$$

$$f = \ln z, \tag{1c}$$

$$f = \sqrt{z}. \tag{1d}$$

2. (**10 points.**) For a given complex number z, say

$$z = \sqrt{2}\, e^{i\frac{\pi}{3}}, \tag{2}$$

evaluate

$$z^2, z^3, z^4, z^5, z^6, z^7, z^8, z^9, z^{10}. \tag{3}$$

Mark all of them on the complex plane. Decipher the pattern.

3. (**10 points.**) (Refer Arfken) The complex quantities

$$a = u + iv, \tag{4a}$$

$$b = x + iy \tag{4b}$$

may also be represented as two-dimensional vectors

$$\mathbf{a} = \hat{\mathbf{x}}\, u + \hat{\mathbf{y}}\, v, \tag{5a}$$

$$\mathbf{b} = \hat{\mathbf{x}}\, x + \hat{\mathbf{y}}\, y. \tag{5b}$$

Show that

$$(a^*)b = \mathbf{a} \cdot \mathbf{b} + i\hat{\mathbf{z}} \cdot \mathbf{a} \times \mathbf{b}. \tag{6}$$

4. (**20 points.**) Find the cube roots of unity by solving the equation

$$z^3 = 1. \tag{7}$$

Mark the points corresponding to the three roots on the complex plane.

5. (**20 points.**) Let

$$z_0 = 2 + i11. \tag{8}$$

Find the three roots the equation

$$z^3 = z_0. \tag{9}$$

Mark the points corresponding to the three roots on the complex plane.

6. (**30 points.**) Find the fifth roots of unity by solving the equation

$$z^5 = 1. \tag{10}$$

Mark the points corresponding to the five roots on the complex plane. Find the five roots of the equation

$$z^5 = -1. \tag{11}$$

Mark the roots on the complex plane. Next, find the roots of the equation

$$z^5 = i \tag{12}$$

and mark the roots on the complex plane. Repeat the exercise for $z^5 = -i$. How do these roots match with the fifth roots of unity?

7. (**20 points.**) Find all z that satisfies the equation

$$e^z = e^{iz}. \tag{13}$$

Show them on a complex plane.