Homework No. 05 (2020 Spring)

PHYS 301: THEORETICAL METHODS IN PHYSICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Wednesday, 2020 Feb 19, 9:00 AM, in class

0. Problems 2 and 4 are to be submitted for assessment. Rest are for practice.

0. Keywords: Curvilinear coordinates, vector calculus.

1. (**Identity.**) Let the unit vectors associated with curvilinear coordinates (ξ_1, ξ_2, ξ_3) be $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ and let (h_1, h_2, h_3) be the respective scale factors. The gradient operator in these coordinates has the form

$$\boldsymbol{\nabla} = \hat{\mathbf{e}}_1 \frac{1}{h_1} \frac{\partial}{\partial \xi_1} + \hat{\mathbf{e}}_2 \frac{1}{h_2} \frac{\partial}{\partial \xi_2} + \hat{\mathbf{e}}_3 \frac{1}{h_3} \frac{\partial}{\partial \xi_3}. \tag{1}$$

A vector field $\mathbf{N}(\xi_1, \xi_2, \xi_3)$ in these coordinates has the form

$$\mathbf{N} = N_1(\xi_1, \xi_2, \xi_3)\, \hat{\mathbf{e}}_1 + N_2(\xi_1, \xi_2, \xi_3)\, \hat{\mathbf{e}}_2 + N_3(\xi_1, \xi_2, \xi_3)\, \hat{\mathbf{e}}_3. \tag{2}$$

The curl in these coordinates can be evaluated as the determinant

$$\boldsymbol{\nabla} \times \mathbf{N} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{\mathbf{e}}_1 & h_2 \hat{\mathbf{e}}_2 & h_3 \hat{\mathbf{e}}_3 \\ \dfrac{\partial}{\partial \xi_1} & \dfrac{\partial}{\partial \xi_2} & \dfrac{\partial}{\partial \xi_3} \\ h_1 N_1 & h_2 N_2 & h_3 N_3 \end{vmatrix}. \tag{3}$$

2. (**20 points.**) Given

$$\mathbf{N} = \hat{\mathbf{z}} \ln \rho \tag{4}$$

and the gradient operator

$$\boldsymbol{\nabla} = \hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \tag{5}$$

evaluate

$$\boldsymbol{\nabla} \times \mathbf{N}. \tag{6}$$

3. (**20 points.**) Given

$$\mathbf{N} = \frac{\hat{\boldsymbol{\phi}}}{\rho} \tag{7}$$

and the gradient operator

$$\boldsymbol{\nabla} = \hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \tag{8}$$

evaluate

$$\boldsymbol{\nabla} \times \mathbf{N}. \tag{9}$$

4. (**20 points.**) Given

$$\mathbf{N} = \hat{\boldsymbol{\phi}} \frac{\rho}{(\rho^2 + z^2)^{\frac{3}{2}}} \tag{10}$$

and the gradient operator

$$\boldsymbol{\nabla} = \hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \tag{11}$$

evaluate

$$\boldsymbol{\nabla} \times \mathbf{N}. \tag{12}$$

5. (**20 points.**) Given

$$\mathbf{N} = \hat{\boldsymbol{\phi}} \frac{\sin \theta}{r^2} \tag{13}$$

and the gradient operator

$$\boldsymbol{\nabla} = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \tag{14}$$

evaluate

$$\boldsymbol{\nabla} \times \mathbf{N}. \tag{15}$$