(Preview of) Midterm Exam No. 01 (2020 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

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1. (**20 points.**) On Ampere law.
2. (**20 points.**) Magnetic vector potential.
3. (**20 points.**) Rotating conductors.
4. (**20 points.**) On elliptic integral.
5. (**20 points.**) A Dirac string constitutes of an infinitely thin solenoid that extends from a point $\mathbf{r}'$ to infinity along an arbitrary curve. Let us consider a Dirac string that extends from the origin to infinity along a straight line, the negative z axis. Thus, we can write the magnetic moment density
$$\hat{\mathbf{z}}\,\lambda\delta(x)\delta(y). \tag{1}$$
   (a) The magnetic vector potential for an infinitesimal element of the 'solenoid' is that of a point magnetic dipole given by
$$d\mathbf{A} = \frac{\mu_0}{4\pi}\frac{d\mathbf{m}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0}{4\pi}\frac{\lambda\rho\,dz'}{[\rho^2+(z-z')^2]^{\frac{3}{2}}}. \tag{2}$$
   Thus, the total magnetic vector potential at any point is obtained by the vector sum of all the elements, using integration,
$$\mathbf{A}(\mathbf{r}) = \int d\mathbf{A} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0}{4\pi}\int_{-\infty}^{0}\frac{\lambda\rho\,dz'}{[\rho^2+(z-z')^2]^{\frac{3}{2}}}. \tag{3}$$
   Complete the integral and show that
$$\mathbf{A}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\,\frac{\mu_0}{4\pi}\frac{\lambda}{\rho}\cot\frac{\theta}{2}. \tag{4}$$
   (b) Evaluate the magnetic field for this magnetic vector potential using the relation
$$\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A} \tag{5}$$
   and show that
$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\lambda\frac{\hat{\mathbf{r}}}{r^2}. \tag{6}$$
   Compare this to the magnetic field due to a magnetic monopole.

   (c) Evaluate
$$\boldsymbol{\nabla}\cdot\mathbf{B}. \tag{7}$$
   Is it zero?