Midterm Exam No. 01 (2020 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale
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1. (**40 points.**) A solenoid has the geometry of a right circular cylinder of radius a and height extending to infinity on both ends. Using Ampere's law show that the magnetic field is uniform inside the solenoid and zero outside the solenoid. How does this result change for a solenoid of arbitrary cross section. Refer literature, and critically go over their arguments. Give a very brief report of your assessment.

2. (**30 points.**) Given
$$\pi(k) = \int_0^{\frac{\pi}{2}} d\psi \frac{1}{(1 - k^2 \sin^2\psi)^{\frac{3}{2}}}. \tag{1}$$
Show that $\pi(k)$ can be expressed in terms of the complete elliptic integrals as
$$\pi(k) = \frac{E(k)}{(1-k^2)}. \tag{2}$$

3. (**30 points.**) A Dirac string constitutes of an infinitely thin solenoid that extends from a point $\mathbf{r}'$ to infinity along an arbitrary curve. Let us consider a Dirac string that extends from the origin to infinity along a straight line, the negative z axis. Thus, we can write the magnetic moment density
$$\hat{\mathbf{z}}\, \lambda \delta(x)\delta(y). \tag{3}$$

   (a) The magnetic vector potential for an infinitesimal element of the 'solenoid' is that of a point magnetic dipole given by
$$d\mathbf{A} = \frac{\mu_0}{4\pi} \frac{d\mathbf{m} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} = \hat{\boldsymbol{\phi}}\, \frac{\mu_0}{4\pi} \frac{\lambda\rho\, dz'}{[\rho^2 + (z - z')^2]^{\frac{3}{2}}}. \tag{4}$$
   Thus, the total magnetic vector potential at any point is obtained by the vector sum of all the elements, using integration,
$$\mathbf{A}(\mathbf{r}) = \int d\mathbf{A} = \hat{\boldsymbol{\phi}}\, \frac{\mu_0}{4\pi} \int_{-\infty}^{0} \frac{\lambda\rho\, dz'}{[\rho^2 + (z - z')^2]^{\frac{3}{2}}}. \tag{5}$$
   Complete the integral and show that
$$\mathbf{A}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\, \frac{\mu_0}{4\pi} \frac{\lambda}{\rho} \cot\frac{\theta}{2}. \tag{6}$$

(b) Evaluate the magnetic field for this magnetic vector potential using the relation

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} \tag{7}$$

and show that

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\lambda\frac{\hat{\mathbf{r}}}{r^2}. \tag{8}$$

Compare this to the magnetic field due to a magnetic monopole.

(c) Evaluate

$$\boldsymbol{\nabla} \cdot \mathbf{B}. \tag{9}$$

Is it zero?