Solutions

Midterm Exam 04 (PHYS-203A) Fall 2020

Problem 1

$$\sin\theta_1 = \sin\theta_2$$

$$\tau = F L \sin\theta$$

Both rods feel same torque, and move with same angular acceleration.

Problem 2

$$\frac{K_T}{K_R} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}I\omega^2} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}\frac{2}{5}Mv^2} = \frac{5}{2}$$

Problem 3

$$a = \frac{(m_1 - m_2)\,g}{\left(m_1 + m_2 + \frac{I}{R^2}\right)}$$

$$I_{disc} < I_{ring}$$

$$a_{disc} > a_{ring}$$

Larger moment of inertia resists rotational motion, and thus the linear motion. Acceleration for ring will be smaller.

Problem 4

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$v = \sqrt{\frac{2gh}{\left(1 + \frac{I}{mR^2}\right)}}$$

Larger moment of inertia resists rotational motion, and the associated linear motion. Hoop will larger radius will move slower, and reach the bottom later.

Problem 5

Angular momentum is conserved, because there is no torque acting about the vertical axis.

Problem 6

Rotational work done by friction cancels the translational work done by friction.

Problem 7

$$\Delta\theta = \frac{\Delta x}{R} = \frac{27}{0.300} = 90.\ \text{rad}$$

$\omega_i = 0$

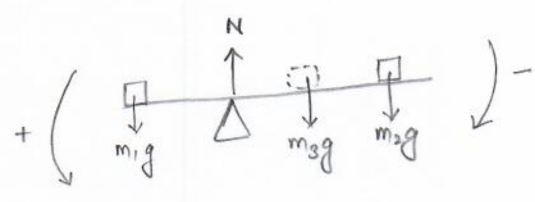
$\omega_f = 75.0\ \frac{\text{rad}}{s}$

$\alpha = ?$

$\Delta t =$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\,\Delta\theta} = \frac{(75.0)^2 - 0^2}{2\,(90.)} = 31\ \frac{\text{rad}}{s^2}$$

Problem 8



$$I\alpha = \tau_N + \tau_1 + \tau_2 + \tau_3$$

$$0 = 0 + m_1 g x_1 - m_2 g x_2 - m_3 g x_3$$

$$x_3 = \frac{m_1 x_1 - m_2 x_2}{m_3}$$

$$= \frac{(1.0)(40.0) - (2.0)(29.0)}{(3.0)}$$

$$= -6.0\ \text{cm}$$

m_3 has to placed 6.0 cm to the left from θ.

Problem 9.

$$I = m_1(0) + m_2(a^2+b^2) + m_3(2b)^2 + m_4(a^2+b^2) + m_0 b^2$$

$$= 0 + 2m(a^2+b^2) + 3m\,4b^2 + 4m(a^2+b^2) + 5mb^2$$

$$= 6ma^2 + 23mb^2 \qquad \Rightarrow \alpha = 6 \quad \text{and} \quad \beta = 23$$

Problem 10

$$\frac{1}{2}m\underbrace{V_A^2}_{=0} + \frac{1}{2}I\underbrace{\omega_A^2}_{=0} + mgh_A = \frac{1}{2}mV_D^2 + \frac{1}{2}I\omega_D^2 + mgh_D$$

$$mg(h_A - h_D) = \frac{1}{2}mV_D^2 + \frac{1}{2}\frac{2}{5}mR^2\omega_D^2 \qquad V_D = \omega_D R$$

$$\cancel{m}g(h_A-h_D) = \frac{7}{10}\cancel{m}\omega_D^2 R^2$$

$$\omega_D = \sqrt{\frac{10}{7}\frac{g(h_A-h_D)}{R^2}}$$

$$= \sqrt{\frac{10}{7}\frac{(9.8)(40.0-10.0)}{(0.15)^2}}$$

$$= 140 \ \frac{\text{rad}}{\text{s}}$$