Homework No. 01 (Fall 2020)

PHYS 520A: ELECTROMAGNETIC THEORY I

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Due date: Monday, 2020 Aug 24, 11.00am

1. (Ref. Schwinger et al., Problem 1, Chapter 1.) Verify the following identities explicitly:

   (a) $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$,

   (b) $\boldsymbol{\nabla} \cdot (\mathbf{A} \times \mathbf{B}) = (\boldsymbol{\nabla} \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\boldsymbol{\nabla} \times \mathbf{B})$,

   (c) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) + \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) + \mathbf{C} \times (\mathbf{A} \times \mathbf{B}) = 0$,

   (d) $\boldsymbol{\nabla} \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A} \times (\boldsymbol{\nabla} \times \mathbf{B}) - \mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{A}) - (\mathbf{A} \times \boldsymbol{\nabla}) \times \mathbf{B} + (\mathbf{B} \times \boldsymbol{\nabla}) \times \mathbf{A}$.

2. For the position vector

$$\mathbf{r} = r\,\hat{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}, \tag{1}$$

   show that

$$\boldsymbol{\nabla} r = \hat{\mathbf{r}}, \quad \boldsymbol{\nabla}\,\mathbf{r} = \mathbf{1}, \quad \boldsymbol{\nabla} \cdot \mathbf{r} = 3, \quad \text{and} \quad \boldsymbol{\nabla} \times \mathbf{r} = 0. \tag{2}$$

   Further, show that for $n \neq 3$

$$\boldsymbol{\nabla}\,\frac{\mathbf{r}}{r^n} = \mathbf{1}\frac{1}{r^n} - \mathbf{r}\,\mathbf{r}\frac{n}{r^{n+2}}, \quad \boldsymbol{\nabla} \cdot \frac{\mathbf{r}}{r^n} = \frac{(3-n)}{r^n}, \quad \text{and} \quad \boldsymbol{\nabla} \times \frac{\mathbf{r}}{r^n} = 0. \tag{3}$$

   For $n = 3$ use divergence theorem to show that

$$\boldsymbol{\nabla} \cdot \frac{\mathbf{r}}{r^n} = 4\pi\,\delta^{(3)}(\mathbf{x}). \tag{4}$$