Homework No. 02 (Fall 2020)

PHYS 520A: ELECTROMAGNETIC THEORY I

Department of Physics, Southern Illinois University–Carbondale

Due date: Friday, 2020 Sep 4, 11.00am

1. (**10 points.**) Show that

$$\boldsymbol{\nabla}(\hat{\mathbf{r}} \cdot \mathbf{a}) = -\frac{1}{r}\,\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{a}) \tag{1}$$

for a uniform (homogeneous in space) vector $\mathbf{a}$.

2. (**10 points.**) Evaluate the number evaluated by the expression

$$\frac{1}{2}\left[\hat{\boldsymbol{\rho}}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial}{\partial\phi}\right] \cdot (\rho\hat{\boldsymbol{\rho}}), \tag{2}$$

where $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\phi}}$ are the unit vectors for cylindrical coordinates (ρ, ϕ) given by

$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}}, \tag{3}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}. \tag{4}$$

3. (**10 points.**) Show that

$$\int_{-\infty}^{\infty} dx\, f(x)\,\delta(x^2 - a^2) = \begin{cases} \frac{f(a)}{|a|}, & \text{if } f(a) \text{ is an even function,} \\ 0, & \text{if } f(a) \text{ is an odd function.} \end{cases} \tag{5}$$

4. (**10 points.**) A uniformly charged infinitely thin disc of radius R and total charge Q is placed on the x-y plane such that the normal vector is along the z axis and the center of the disc at the origin. Write down the charge density of the disc in terms of δ-function(s). Integrate over the charge density and verify that it returns the total charge on the disc.