# Homework No. 08 (Fall 2020)

**PHYS 520A: ELECTROMAGNETIC THEORY I**

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Due date: Friday, 2020 Oct 30, 11.00am

1. (**20 points.**) Consider a solid right circular cylinder of radius $R$, with axis along the $z$ axis and of infinite length, with uniform permanent polarization

$$\mathbf{P}(\mathbf{r},t) = \mathbf{P}_0\,\theta(R-\rho), \tag{1}$$

where $\rho^2 = x^2 + y^2$ and $\mathbf{P}_0$ is perpendicular to the axis of the cylinder.

   (a) Show that the effective charge density is given by the expression

$$\rho_{\text{eff}}(\mathbf{r}) = -\boldsymbol{\nabla}\cdot\mathbf{P} = \mathbf{P}_0\cdot\hat{\boldsymbol{\rho}}\,\delta(\rho - R). \tag{2}$$

   (b) Discuss the case when $\mathbf{P}_0$ is parallel to the axis of the cylinder. Further, qualitatively, discuss the case if, in addition, the cylinder was of finite length in the direction of $z$.

2. (**20 points.**) Consider a radially polarized sphere of radius $R$ described by

$$\mathbf{P}(\mathbf{r}) = \alpha\,\mathbf{r}\,\theta(R-r), \tag{3}$$

where $\alpha$ is constant.

   (a) Calculate $-\boldsymbol{\nabla}\cdot\mathbf{P}$. Thus, find the effective charge density to be

$$\rho_{\text{eff}} = -3\alpha\theta(R-r) + \alpha r\delta(r-R). \tag{4}$$

   (b) Using

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho_{\text{eff}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}, \tag{5}$$

   evaluate the electric potential to be

$$\phi(\mathbf{r}) = \begin{cases} -\dfrac{\alpha}{2\varepsilon_0}(R^2 - r^2), & r < R, \\ 0, & R < r. \end{cases} \tag{6}$$

   (Hint: Choose observation point $\mathbf{r}$ along $\hat{\mathbf{z}}$.)

   (c) Evaulate the electric field

$$\mathbf{E}(\mathbf{r}) = -\boldsymbol{\nabla}\phi(\mathbf{r}) = \begin{cases} -\dfrac{\alpha}{\varepsilon_0}\,\mathbf{r}, & r < R, \\ 0, & r > R. \end{cases} \tag{7}$$

(d) Find the enclosed charge inside a sphere of radius $r$ using

$$Q_{\text{en}} = \int d^3r' \, \rho_{\text{eff}}(\mathbf{r}') \tag{8}$$

for $r < R$ and $r > R$.

(e) Use Gauss's law,

$$\oint d\mathbf{a} \cdot \mathbf{E} = \frac{1}{\varepsilon_0} Q_{\text{en}}, \tag{9}$$

to verify the expression for the electric field in Eq. (7).

(f) Interpret the electric field for $r > R$ as the electric field due to the total charge inside $r \leq R$.