Homework No. 10 (Fall 2020)

PHYS 520A: ELECTROMAGNETIC THEORY I

Department of Physics, Southern Illinois University–Carbondale

~~Due date: Friday, 2020 Dec 4, 11.00am~~

Due date: None

1. (**20 points.**) Show that the speed of energy flow of a monochromatic electromagnetic wave in a dispersive medium (for slowly evolving field) when both ε and μ are frequency dependent is given by

$$\frac{v_E}{c} = \left[\frac{d}{d\omega}\left(\omega\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}\right)\right]^{-1}. \tag{1}$$

   Determine the speed of energy flow for the case

$$\mu = \mu_0 \qquad \text{and} \qquad \frac{\varepsilon}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\omega^2} \tag{2}$$

   to be

$$\frac{v_E}{c} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} < 1. \tag{3}$$

2. (**20 points.**) The constitutive relations in a nondispersive media are

$$\mathbf{D} = \varepsilon\mathbf{E}, \tag{4a}$$
$$\mathbf{B} = \mu\mathbf{H}, \tag{4b}$$

   where ε and μ are constants. The ratio of speed of light in vacuum c to speed of light in the medium v is the refractive index of the medium

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}. \tag{5}$$

   The theory of relativity states that velocity of energy flow can not be larger than the speed of light in vacuum. Thus, $n > 1$. Let $\mu = \mu_0$. Consider the dielectric model

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma - \omega^2}. \tag{6}$$

   This is a complex number, which means a complex velocity of propagation v and a complex index of refraction

$$n = n_r + in_i = \frac{c}{v} = \sqrt{\frac{\varepsilon(\omega)}{\varepsilon_0}}. \tag{7}$$

A complex refractive index signifies that the propagation is accompanied by absorption

$$e^{-i\omega\left(t-\frac{x}{v}\right)} = e^{-i\omega\left(t-n\frac{x}{c}\right)} = e^{-n_i\frac{\omega}{c}x}e^{-i\omega\left(t-n_r\frac{x}{c}\right)}. \tag{8}$$

Thus, c/n_r plays the role of phase velocity and $n_i\omega/c$ is a coefficient of absorption. Plot n_r as a function of ω and verify that it crosses the line $n = 1$ near $\omega = \omega_0$. Thus, apparently, signal in a dispersive medium violates causality. This contradiction was resolved by Sommerfeld and Brillouin in 1914. Translated versions of their papers have been published in a book titled 'Wave Propagation and Group Velocity' by Brillouin in 1960. The book is available at https://archive.org. Very briefly present the resolution here.