Homework No. 02 (2021 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2021 Feb 2, 12.30pm

1. (**20 points.**) The vector potential for a straight wire of infinite extent carrying a steady current I is
$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0}{4\pi}\hat{\mathbf{z}}\, 2I \ln\frac{\rho}{2L}, \qquad \rho \ll L, z \ll L, \tag{1}$$
where L is understood to be sufficiently larger than ρ and z (or $L \to \infty$) in the equation. Note that the restriction $\rho \ll L$, and $z \ll L$, is required to be consistent with $\boldsymbol{\nabla}\cdot\mathbf{j} = 0$. The magnetic field around the wire is given by
$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi\rho}. \tag{2}$$
Starting from
$$\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A} \tag{3}$$
derive the relation for the flux of magnetic field
$$\Phi = \int_S d\mathbf{a}\cdot\mathbf{B} = \oint d\mathbf{l}\cdot\mathbf{A}. \tag{4}$$
Consider the loop to constitute a rectangle in the constant ϕ plane with $\rho_1 < \rho < \rho_2 < \infty$ and $-\infty < z_1 < z < z_2 < \infty$. Show that
$$\Phi = \frac{\mu_0}{4\pi} 2Ih \ln\frac{\rho_2}{\rho_1}, \tag{5}$$
where $h = z_2 - z_1$. What is the implication of the observation that the surface enclosing a closed curve is not unique. (Do not extend the surface to infinity to remain consistent with $\boldsymbol{\nabla}\cdot\mathbf{j} = 0$.)

2. (**20 points.**) The magnetic field for a straight wire of infinite extent carrying a steady current I is given by
$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi\rho}. \tag{6}$$
Verify that
$$\boldsymbol{\nabla}\cdot\mathbf{B} = 0 \tag{7}$$
everywhere. In particular, investigate if the magnetic field is divergenceless on the wire, where $\rho = 0$. Next, evaluate
$$\boldsymbol{\nabla}\times\mathbf{B} \tag{8}$$
everywhere. Thus, check if the magnetic field due to a straight current carrying wire satisfies the two Maxwell equations relevant for magnetostatics.