Solutions

PHYS - 205A-001 (Midterm Exam 03) Spring 2022

Problem 1

When the work done by a force is independent of the path it is called a conservative force. Examples are gravitational force and force due to a spring.

Problem 2

Force. $F = -\frac{\partial U}{\partial x}$.

Problem 3

Energy is reduced when center of mass moves vertically down. For the object, in the position in figure, the center of mass is above R. Thus, it is unstable.

Problem 4

$$K_A + U_A^g + U_A^s = K_c + U_c^g + U_c^s.$$

$$\frac{1}{2}mv_A^2 + mgh_A + \frac{1}{2}kx_A^2 = \frac{1}{2}mv_c^2 + mgh_c + \frac{1}{2}kx_c^2$$

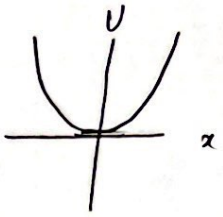
($\frac{1}{2}mv_A^2 = 0$, $\frac{1}{2}kx_A^2 = 0$, $\frac{1}{2}mv_c^2 = 0$, $mgh_c = 0$)

$$mgh_A = \frac{1}{2}kx_c^2$$

$$U_c^s = \frac{1}{2}kx_c^2 = mgh_A = (20.0)(9.8)\,1.0 = 196\ \text{J}$$

$$= 2.0 \times 10^2\ \text{J}$$

Problem 5



(a) $U = 0 \Rightarrow ax^2 + bx^4 = 0$

$x^2(a + bx^2) = 0$

$\Rightarrow x = 0$ (since $a + bx^2 \geq 0$)

(b) $F = -\frac{\partial U}{\partial x} = -2ax - 4bx^3$

$F = 0 \Rightarrow -2x(a + 2bx^2) = 0$

$\Rightarrow x = 0$ (since $a + 2bx^2 > 0$)

(c) $\frac{\partial^2 U}{\partial x^2} = 2a + 12bx^2$

$\left.\frac{\partial^2 U}{\partial x^2}\right|_{x=0} = 2a > 0 \Rightarrow x = 0$ is a stable point.

Problem 6

$m_1 = 100.\,kg$

$m_2 = 1.0\,kg$

$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_{1i} + 0 = \left(\frac{100 - 1.0}{100 + 1.0}\right) 10. = 9.8\,\frac{m}{s}$

$V_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) V_{1i} + 0 = \frac{2(100.)}{100 + 1.0}\, 10. = 19.8\,\frac{m}{s} = 20.\,\frac{m}{s}$

Problem 7

$$x_{cm} = \frac{\int_0^L dx \; x \; \rho(x)}{\int_0^L dx \; \rho(x)} = \frac{\int_0^L dx \; x \; a}{\int_0^L dx \; a} = \frac{a\frac{L^2}{2}}{aL} = \frac{L}{2}.$$