Midterm Exam No. 01 (2022 Spring)

PHYS 510: CLASSICAL MECHANICS

Department of Physics, Southern Illinois University-Carbondale
Date: 2022 Feb 17

1. (20 points.) Evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)} \tag{1}$$

of the following functional,

$$F[u] = \int_{a}^{b} dx \left[1 + u \left(\frac{du}{dx} \right) \right], \tag{2}$$

assuming no variation at the end points.

2. (20 points.) Evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)} \tag{3}$$

of the following functional,

$$F[u] = \int_{a}^{b} dx \left[1 + u \left(\frac{d^{2}u}{dx^{2}} \right) \right], \tag{4}$$

assuming no variation at the end points.

3. (20 points.) Evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)} \tag{5}$$

of the following functional,

$$F[u] = \int_a^b dx \, \frac{d}{dx} \sqrt{1 + \frac{du}{dx} + \frac{d^2u}{dx^2}},\tag{6}$$

assuming no variation at the end points.

4. (20 points.) Prove the intuitively obvious statement that the curve of shortest distance going through two points on a plane, the geodesics of a plane, are straight lines passing through the two points.