

Homework No. 03 (Fall 2022)

PHYS 205B: UNIVERSITY PHYSICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2022 Sep 27, 9:30 AM, on D2L

Instructions

- You are encouraged to use any of the resources to complete this homework. However, the extent to which you depend on resources while doing this homework is a measure of how much extra work you need to put in to master the associated concepts. Solutions should be the last resource and are available at

<http://sphics.com/tc/202108-SIU-P205B/>.

- Describe your thought process in detail and organize it clearly. Make sure your answer has units and the right number of significant digits.
- After completion, scan the pages as a single PDF file, and submit the file on D2L (under Assesments → Assignments).

Problems

1. (**10 points.**) A sphere with uniform charge distribution $-Q = -3.0 \mu\text{C}$ is fixed at the origin. Point A is on a sphere of radius 5.0 cm and point B is on a sphere of radius 10.0 cm. Refer Figure 1.

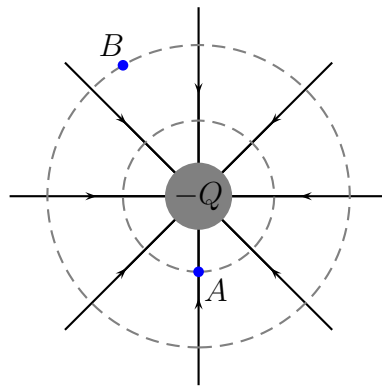


Figure 1: Problem 1

- (a) What is the work done by the electric force acting on charge $q = +2.0 \mu\text{C}$, when q is moved from point A to point B .
- (b) What is the change in the electric potential energy between $-Q$ and q when q is moved from point A to point B .
- (c) If there are no other forces acting on charge q , using the work-energy theorem calculate the change in kinetic energy of charge q .

Solution

2. (10 points.) A cylinder with uniform line-charge distribution $\lambda = +5.0 \mu\text{C}/\text{m}$ is fixed at the origin. Point A is on a cylinder of radius 10.0 cm and point B is on a cylinder of radius 20.0 cm. Refer Figure 2.

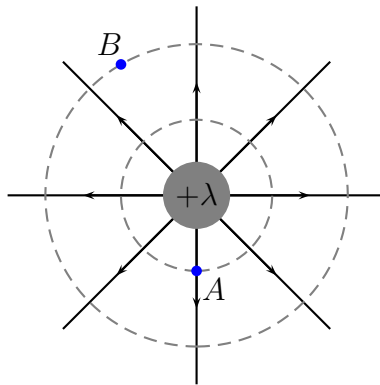


Figure 2: Problem 2

- (a) What is the work done by the electric force acting on charge $q = +2.0 \mu\text{C}$, when q is moved from point A to point B .
- (b) What is the change in the electric potential energy between the line-charge λ and q when q is moved from point A to point B .
- (c) If there are no other forces acting on charge q , using the work-energy theorem calculate the change in kinetic energy of charge q .

Solution

3. (10 points.) A positive charge $Q_1 = 1.0 \text{ nC}$ is held fixed. Another positive charge $Q_2 = 2Q_1$ is tied to charge Q_1 using a string of length $a = 5.0 \text{ cm}$. Assume the radius of the two charges to be small in comparison to a . The charges have masses $m_1 = 0.05 \text{ grams}$ and $m_2 = 2m_1$. When the string is cut the two charges fly off in opposite directions. Determine the speed of each of the charges when they are (infinitely) far apart. (Hint: Use conservation of momentum and conservation of energy.)

Solution

4. (10 points.) Determine the total energy required to assemble four identical positive charges Q at the corners of a square of length L . Assume that the charges are brought from infinity.

Solution

5. (10 points.) Four charges $q_1 = q$, $q_2 = -2q$, $q_3 = -3q$, and $q_4 = 4q$, are placed at the corners of a square of side L , such that q_1 and q_4 are at diagonally opposite corners. Refer Figure 3.

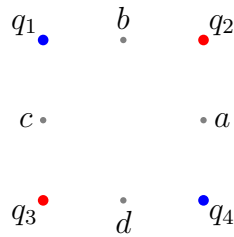


Figure 3: Problem 5

- What is the electric potential at the center of square?
- What is the electric potential at point a ?
- What is the electric potential at point b ?
- What is the electric potential difference between points a and c ?
- How much potential energy is required to move another charge q from infinity to the center of the square?
- How much additional potential energy is required to move this charge from the center of the square to point a ?

Solution

6. (10 points.) The electric dipole moment of a configuration consisting of two equal and opposite point charges, separated by a distance d , is defined to be

$$\mathbf{p} = q\mathbf{d}, \tag{1}$$

where \mathbf{d} points from the negative to the positive charge and $d = |\mathbf{d}|$. Let $d = 2a$. The electric potential of the electric dipole at the point (x, y, z) is given by the expression

$$V(x, y, z) = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{(x^2 + y^2 + (z - a)^2)}} - \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{(x^2 + y^2 + (z + a)^2)}}. \tag{2}$$

The electric field of the electric dipole can be calculated using

$$\mathbf{E}(x, y, z) = -\nabla V \tag{3a}$$

$$= -\hat{\mathbf{i}} \frac{\partial V}{\partial x} - \hat{\mathbf{j}} \frac{\partial V}{\partial y} - \hat{\mathbf{k}} \frac{\partial V}{\partial z}. \tag{3b}$$

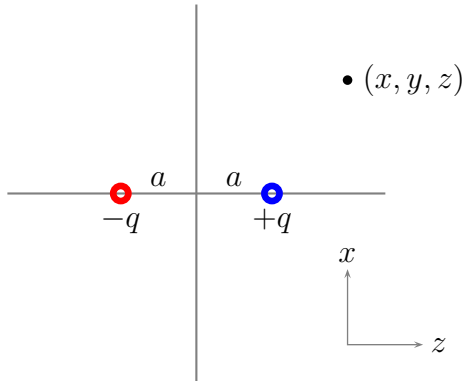


Figure 4: Problem 6

- Calculate the electric field on the x axis.
- Determine the electric field on the x axis for $a \ll x$.
- Calculate the electric field on the z axis.
- Determine the electric field on the z axis for $a \ll z$.

Solution

7. (10 points.) Consider a uniformly charged disc of radius R with charge per unit area σ placed on the yz plane such that the origin is the center of the disc. Determine the electric potential on the x axis to be

$$V(x) = -\frac{\sigma}{2\epsilon_0} \left[x - \sqrt{x^2 + R^2} \right]. \quad (4)$$

Using the fact that the electric field is the negative gradient of the electric potential, calculate the x -component of the electric field on the x axis to be

$$E_x = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]. \quad (5)$$

- Show that this leads to the potential and electric field of a point charge in the limit $R \ll x$.
- Analyze the limit $x \ll R$. Show that this leads to the potential and electric field for a non-conducting uniformly charged plate. Plot the electric potential as a function of x for this case.

Solution

8. (10 points.) The electric field inside and outside a conducting sphere of radius R is given by

$$\mathbf{E} = \begin{cases} 0, & r < R \text{ (inside)}, \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}}, & R < r \text{ (outside)}. \end{cases} \quad (6)$$

Determine the electric potential inside and outside the sphere using

$$\Delta V = - \int_{\mathbf{r}_i}^{\mathbf{r}_f} d\mathbf{l} \cdot \mathbf{E}. \quad (7)$$

Hint: Since the electric field is zero inside a perfect conductor, the electric potential inside the conductor must be a constant.

Solution