

Equation Sheet for PHYS-205B University Physics

(Last updated: December 7, 2021)
This list will evolve during the semester.

1. Electrostatic force and electric field:

(a) Charge and masses:

$$e = 1.60 \times 10^{-19} \text{ C}, \quad m_e = 9.11 \times 10^{-31} \text{ kg}, \quad m_p = 1.67 \times 10^{-27} \text{ kg}. \quad (1)$$

(b) Coulomb's law:

$$\vec{\mathbf{F}} = \frac{kq_1q_2}{r^2} \hat{\mathbf{r}}, \quad k = \frac{1}{4\pi\epsilon_0}, \quad k = 8.99 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}, \quad \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N m}^2}. \quad (2)$$

(c) Electric field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}. \quad (3)$$

(d) Electric fields for some relevant geometries:

Point : $\vec{\mathbf{E}} = \hat{\mathbf{r}} \frac{kQ}{r^2},$ (4a)

Line : $\vec{\mathbf{E}} = \hat{\mathbf{r}} \frac{2k\lambda}{r}, \quad \lambda = \frac{Q}{L},$ (4b)

(Dielectric) Plane : $\vec{\mathbf{E}} = \hat{\mathbf{r}} \frac{\sigma}{2\epsilon_0} = \hat{\mathbf{r}} 2\pi k\sigma, \quad \sigma = \frac{Q}{A},$ (4c)

(Conducting) Plane : $\vec{\mathbf{E}} = \hat{\mathbf{r}} \frac{\sigma}{\epsilon_0}, \quad \sigma = \frac{Q}{A}.$ (4d)

(4e)

(e) Gauss' law:

$$\Phi_E = \oint_S d\vec{\mathbf{a}} \cdot \vec{\mathbf{E}} = \frac{Q_{en}}{\epsilon_0}. \quad (5)$$

2. Electric potential energy and electric potential:

(a) Electric potential

$$\Delta U = - \int_a^b \mathbf{F} \cdot d\mathbf{l}, \quad \Delta V = - \int_a^b \mathbf{E} \cdot d\mathbf{l}, \quad \Delta U = q\Delta V. \quad (6)$$

Point charge: $U = \frac{kq_1q_2}{r}, \quad V = \frac{kq}{r},$ (7a)

Constant field: $U = -\mathbf{F} \cdot \mathbf{d}, \quad V = -\mathbf{E} \cdot \mathbf{d}.$ (7b)

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(d) Ampere's law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{en}. \quad (17)$$

5. Faraday's law of induction:

(a) Magnetic flux:

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}. \quad (18)$$

(b) Induced voltage:

$$V_{eff} = -N \frac{d\Phi_B}{dt}. \quad (19)$$

Electric generator: $V_{eff} = NAB\omega \sin \omega t.$

Transformer: $\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}.$

(c) Inductance:

$$V = L \frac{dI}{dt}, \quad L = \mu_0 n^2 AL, \quad U = \frac{1}{2} LI^2, \quad U = \frac{1}{2\mu_0} B^2. \quad (20)$$

(d) RL circuits: time constant $= \tau = L/R.$

$$I(t) = \frac{V}{R} \left(1 - e^{-t/\tau}\right) \quad (\text{charging}), \quad (21a)$$

$$I(t) = \frac{V}{R} e^{-t/\tau} \quad (\text{discharging}). \quad (21b)$$

6. Electromagnetic waves:

(a) Speed of light: $c = 2.998 \times 10^8 \text{ m/s}.$

(b) Energy density: $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2.$

(c) Doppler effect: $f' = f \left(1 \pm \frac{v_{rel}}{c}\right),$ for $v_{rel} \ll c.$

(d) Polarization: $I' = I \cos^2 \theta.$

7. Ray optics:

(a) General equations:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \quad R = 2f, \quad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \quad (22)$$

(b) Refraction:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2, \quad n = \frac{c}{v}, \quad c = 3 \times 10^8 \text{ m/s}. \quad (23)$$

(b) Capacitance:

$$C = \frac{Q}{V}, \quad U = \frac{1}{2} QV, \quad U = \frac{1}{2} \epsilon_0 E^2. \quad (8)$$

Parallel-plate capacitor:

$$E = \frac{Q}{\epsilon_0 A}, \quad C = \frac{\epsilon_0 A}{d}. \quad (9)$$

3. DC circuits:

(a) Current: $I = \frac{Q}{\Delta t},$ Resistance: $R = \frac{\rho l}{A}, \quad \rho - \rho_0 = \alpha \rho_0 (T - T_0),$

(b) Ohm's law: $V = IR,$ Power: $P = IV = I^2 R = \frac{V^2}{R}.$

(c) Resistors:

$$R_{tot} = R_1 + R_2 \quad (\text{series}) \quad (10a)$$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (\text{parallel}) \quad (10b)$$

(d) Capacitors:

$$C_{tot} = C_1 + C_2 \quad (\text{parallel}) \quad (11a)$$

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} \quad (\text{series}) \quad (11b)$$

(e) RC circuits: time constant, $\tau = RC.$

$$Q(t) = VC \left(1 - e^{-t/\tau}\right) \quad (\text{charging}), \quad (12a)$$

$$Q(t) = Q_0 e^{-t/\tau} \quad (\text{discharging}). \quad (12b)$$

4. Magnetostatics:

(a) Magnetic force:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}, \quad \vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}. \quad (13)$$

Right hand rule: Index- $v/I,$ Middle- $B,$ Thumb- $F.$

(b) Torque on a magnetic dipole moment:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}, \quad \vec{\boldsymbol{\mu}} = NIA\hat{\mathbf{n}}. \quad (14)$$

(c) Magnetic fields for some relevant configurations:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/Amp}. \quad (15)$$

Straight wire segment : $\vec{\mathbf{B}} = \hat{\phi} \frac{\mu_0 I}{4\pi r} (\sin \theta_1 + \sin \theta_2),$ (16a)

Circular segment of wire : $\vec{\mathbf{B}} = \hat{z} \frac{\mu_0 I}{4\pi R} \theta,$ (16b)

Solenoid : $\vec{\mathbf{B}} = \hat{z} \mu_0 In, \quad n = N/L.$ (16c)

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Equation Sheet for PHYS-205A University Physics

(Last updated: January 18, 2016)
This list will evolve during the semester.

1. General mathematics:

(a) Units:

$$c = 10^{-2}, \quad m = 10^{-3}, \quad \mu = 10^{-6}, \quad n = 10^{-9}, \quad p = 10^{-12}. \quad (1a)$$

$$d = 10^2, \quad k = 10^3, \quad M = 10^6, \quad G = 10^9, \quad T = 10^{12}. \quad (1b)$$

(b) Geometry of a right triangle:

$$\sin \theta = \frac{\text{opp. to angle}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opp. to angle}}{\text{adj to angle}}, \quad (2a)$$

$$\cos \theta = \frac{\text{adj. to angle}}{\text{hypotenuse}}, \quad A^2 = A_x^2 + A_y^2. \quad (2b)$$

(c) Quadratic equation:

$$ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (3)$$

(d) Calculus:

$$\frac{d}{dx}x^n = nx^{n-1}, \quad \int x^n dx = \frac{x^{n+1}}{n+1}. \quad (4)$$

2. Kinematic equations:

(a) Velocity and acceleration:

$$v(t) = \frac{dx}{dt}, \quad a(t) = \frac{dv}{dt}. \quad (5)$$

(b) Uniform velocity ($a = 0$):

$$\Delta x = v\Delta t. \quad (6)$$

(c) Uniform acceleration:

$$v_f = v_i + a\Delta t; \quad \Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2; \quad v_f^2 = v_i^2 + 2a\Delta x; \quad (7a)$$

$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}; \quad \Delta x = v_f\Delta t - \frac{1}{2}a\Delta t^2. \quad (7b)$$

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(e) Mechanical energy:

$$\Delta K + \Delta U_g + \Delta U_s = W_{\text{fric}} + \dots \quad (18)$$

5. Linear momentum:

$$\vec{p} = m\vec{v}, \quad \vec{p}_f - \vec{p}_i = \vec{F}\Delta t. \quad (19)$$

(a) Conservation of linear momentum:

$$\vec{p}_{1i} + \vec{p}_{2i} + \dots = \vec{p}_{1f} + \vec{p}_{2f} + \dots \quad (20)$$

(b) Elastic collision of two objects:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}, \quad (21a)$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}. \quad (21b)$$

(c) Center of mass:

$$X_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + \dots}{m_1 + m_2 + \dots} \rightarrow \int x \frac{dm}{dm}. \quad (22)$$

6. Rotational dynamics:

(a) Kinematic equations:

i. Constant angular speed ($\alpha = 0$): $\Delta\theta = \omega\Delta t$.

ii. Constant angular acceleration:

$$\omega_f = \omega_i + \alpha\Delta t, \quad \Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2, \quad (23)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta, \quad \Delta\theta = \omega_f\Delta t - \frac{1}{2}\alpha\Delta t^2, \quad \frac{\Delta\theta}{\Delta t} = \frac{\omega_i + \omega_f}{2}. \quad (24)$$

(b) Rotational inertia (moment of inertia): $I = \int r^2 dm$.

Point mass, distance R from axis : $I = MR^2$ (25a)

circular Ring, axis perpendicular to plane of ring : $I = MR^2$ (25b)

Spherical shell, about diameter : $I = \frac{2}{3}MR^2$ (25c)

Solid cylinder, about cylinder axis : $I = \frac{1}{2}MR^2$ (25d)

Solid sphere, about diameter : $I = \frac{2}{5}MR^2$ (25e)

(c) Torque:

$$\tau = RF \sin \theta, \quad \vec{\tau} = \frac{d\vec{L}}{dt}. \quad (26)$$

(d) Time of flight, horizontal range, and maximum height in projectile motion:

$$\Delta T = \frac{2v_0 \sin \theta_0}{g}, \quad R = \frac{v_0^2 \sin 2\theta_0}{g}, \quad H = \frac{v_0^2 \sin^2 \theta_0}{2g}. \quad (8)$$

(e) Relative velocity: $\vec{v}_{AB} = \vec{v}_{AG} + \vec{v}_{GB}$.

3. Forces:

(a) Newton's law:

$$\vec{F}_1 + \vec{F}_2 + \dots = m\vec{a} \quad (9)$$

(b) Gravitational force:

$$F_G = \frac{Gm_1m_2}{R^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2. \quad (10)$$

(c) Force due to friction:

$$F_f \begin{cases} \leq \mu_s N, & (\text{static case}), \\ = \mu_k N, & (\text{kinetic case}). \end{cases} \quad (11)$$

(d) Circular motion:

$$v = \omega r, \quad \omega = 2\pi f, \quad f = \frac{1}{T}, \quad (12)$$

$$a_c = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r = \frac{4\pi^2}{T^2} r \quad (13)$$

(e) Resistive forces: $R = bv$ (for small speeds) and $R = \frac{1}{2}D\rho Av^2$ (for high speeds).

4. Work and energy:

(a) Kinetic energy:

$$K = \frac{1}{2}mv^2 \quad (14)$$

(b) Work done by a force:

$$W = \int \mathbf{F} \cdot d\mathbf{l} \rightarrow Fd \cos \theta \quad (15)$$

(c) Work-kinetic energy theorem:

$$W_1 + W_2 + \dots = \Delta K \quad (16)$$

(d) Potential energies:

$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2. \quad (17)$$

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(d) Rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2}I\omega^2. \quad (27)$$

(e) Angular momentum:

$$L = I\omega, \quad \vec{L} = \vec{r} \times \vec{p}. \quad (28)$$

7. Gravitation:

$$F_G = \frac{Gm_1m_2}{R^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2. \quad (29)$$

8. Waves and oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}, \quad \lambda = \frac{2\pi}{k}, \quad v = \lambda f. \quad (30)$$

(a) Simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}. \quad (31)$$

(b) Mass-spring system:

$$T = 2\pi\sqrt{\frac{m}{k}}. \quad (32)$$

9. Fluid dynamics:

(a) Density-mass relation:

$$m = \rho V. \quad (33)$$

Density of water = 10^3 kg/m^3 .

(b) Pressure:

$$P = \frac{F}{A}. \quad (34)$$

(c) Pressure in a static fluid:

$$P_2 = P_1 + \rho gh. \quad (35)$$

1 atm = $1.01 \times 10^5 \text{ N/m}^2$.

(d) Buoyant force:

$$B = m_{\text{dis}}g, \quad (36)$$

where $m_{\text{dis}} = \rho_{\text{dis}}V_{\text{dis}}$ is the mass of the displaced liquid.

(e) Continuity equation:

$$A_1v_1 = A_2v_2. \quad (37)$$

(f) Bernoulli's equation:

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{const} \quad (38)$$