Equation Sheet for PHYS-205B University Physics

(Last updated: January 16, 2023)
This list will evolve during the semester.

1. Electrostatic force and electric field:

   (a) Charge and masses:

   $$e = 1.60 \times 10^{-19}\,\mathrm{C}, \quad m_e = 9.11 \times 10^{-31}\,\mathrm{kg}, \quad m_p = 1.67 \times 10^{-27}\,\mathrm{kg}. \tag{1}$$

   (b) Coulomb's law:

   $$\vec{\mathbf{F}} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}}, \quad k = \frac{1}{4\pi\varepsilon_0}, \quad k = 8.99 \times 10^9 \frac{\mathrm{N\,m^2}}{\mathrm{C^2}}, \quad \varepsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C^2}}{\mathrm{N\,m^2}}. \tag{2}$$

   (c) Electric field:

   $$\vec{\mathbf{F}} = q\,\vec{\mathbf{E}}. \tag{3}$$

   (d) Electric fields for some relevant geometries:

   $$\text{Point}: \quad \vec{\mathbf{E}} = \hat{\mathbf{r}}\,\frac{kQ}{r^2}, \tag{4a}$$

   $$\text{Line}: \quad \vec{\mathbf{E}} = \hat{\mathbf{r}}\,\frac{2k\lambda}{r}, \qquad \lambda = \frac{Q}{L}, \tag{4b}$$

   $$\text{(Dielectric) Plane}: \quad \vec{\mathbf{E}} = \hat{\mathbf{r}}\,\frac{\sigma}{2\varepsilon_0} = \hat{\mathbf{r}}\,2\pi k\sigma, \qquad \sigma = \frac{Q}{A}, \tag{4c}$$

   $$\text{(Conducting) Plane}: \quad \vec{\mathbf{E}} = \hat{\mathbf{r}}\,\frac{\sigma}{\varepsilon_0}, \qquad \sigma = \frac{Q}{A}. \tag{4d}$$

   $$\tag{4e}$$

   (e) Gauss' law:

   $$\Phi_E = \oint_S d\vec{\mathbf{a}} \cdot \vec{\mathbf{E}} = \frac{Q_{\mathrm{en}}}{\varepsilon_0}. \tag{5}$$

2. Electric potential energy and electric potential:

   (a) Electric potential

   $$\Delta U = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot d\mathbf{l}, \qquad \Delta V = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l}, \qquad \Delta U = q\Delta V. \tag{6}$$

   $$\text{Point charge:} \quad U = \frac{kq_1q_2}{r}, \qquad V = \frac{kq}{r}. \tag{7a}$$

   $$\text{Constant field:} \quad U = -\mathbf{F} \cdot \mathbf{d}, \qquad V = -\mathbf{E} \cdot \mathbf{d}. \tag{7b}$$

   (b) Capacitance:

   $$C = \frac{Q}{V}, \qquad U = \frac{1}{2}QV, \qquad U = \frac{1}{2}\varepsilon_0 E^2. \tag{8}$$

   Parallel-plate capacitor:

   $$E = \frac{Q}{\varepsilon_0 A}, \qquad C = \frac{\varepsilon_0 A}{d}. \tag{9}$$

3. DC circuits:

   (a) Current: $I = \dfrac{Q}{\Delta t}$, Resistance: $R = \dfrac{\rho l}{A}$, $\rho - \rho_0 = \alpha\rho_0(T - T_0)$,

   (b) Ohm's law: $V = IR$, Power: $P = IV = I^2R = \dfrac{V^2}{R}$.

   (c) Resistors:

   $$R_{\mathrm{tot}} = R_1 + R_2 \qquad \text{(series)} \tag{10a}$$

   $$\frac{1}{R_{\mathrm{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{(parallel)} \tag{10b}$$

   (d) Capacitors:

   $$C_{\mathrm{tot}} = C_1 + C_2 \qquad \text{(parallel)} \tag{11a}$$

   $$\frac{1}{C_{\mathrm{tot}}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{(series)} \tag{11b}$$

   (e) RC circuits: time constant, $\tau = RC$.

   $$Q(t) = VC\left(1 - e^{-\frac{t}{RC}}\right) \qquad \text{(charging)}, \tag{12a}$$

   $$Q(t) = Q_0 e^{-\frac{t}{RC}} \qquad \text{(discharging)}. \tag{12b}$$

4. Magnetostatics:

   (a) Magnetic force:

   $$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}, \quad \vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}. \tag{13}$$

   Right hand rule: Index-v/I, Middle-B, Thumb-F.

   (b) Torque on a magneic dipole moment:

   $$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}, \quad \vec{\boldsymbol{\mu}} = NIA\,\hat{\mathbf{n}}. \tag{14}$$

   (c) Magnetic fields for some relevant configurations:

   $$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/Amp}. \tag{15}$$

   $$\text{Straight wire segment}: \quad \vec{\mathbf{B}} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2), \tag{16a}$$

   $$\text{Circular segment of wire}: \quad \vec{\mathbf{B}} = \hat{\mathbf{z}}\,\frac{\mu_0 I}{4\pi R}\theta, \tag{16b}$$

   $$\text{Solenoid}: \quad \vec{\mathbf{B}} = \hat{\mathbf{z}}\,\mu_0 In, \quad n = N/L. \tag{16c}$$

   (d) Ampere's law:

   $$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\mathrm{en}}. \tag{17}$$

5. Faraday's law of induction:

   (a) Magnetic flux:

   $$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}. \tag{18}$$

   (b) Induced voltage:

   $$V_{\mathrm{eff}} = -N\frac{d\Phi_B}{dt}. \tag{19}$$

   Electric generator: $V_{\mathrm{eff}} = NAB\omega\sin\omega t$.
   Transformer: $\dfrac{V_p}{V_s} = \dfrac{N_p}{N_s} = \dfrac{I_s}{I_p}$.

   (c) Inductance:

   $$V = L\frac{dI}{dt}, \quad L - \mu_o n^2 Al, \quad U = \frac{1}{2}LI^2, \quad U = \frac{1}{2\mu_0}B^2. \tag{20}$$

   (d) RL circuits: time constant $= \tau = L/R$.

   $$I(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{L/R}}\right) \qquad \text{(charging)}, \tag{21a}$$

   $$I(t) = \frac{V}{R}e^{-\frac{t}{L/R}} \qquad \text{(discharging)}. \tag{21b}$$

6. Electromagnetic waves:

   (a) Speed of light: $c = 2.998 \times 10^8\,\mathrm{m/s}$.

   (b) Energy density: $u = \dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{1}{2\mu_0}B^2 = \varepsilon_0 E^2 = \dfrac{1}{\mu_0}B^2$.

   (c) Doppler effect: $f' = f\left(1 \pm \dfrac{v_{\mathrm{rel}}}{c}\right)$, for $v_{\mathrm{rel}} \ll c$.

   (d) Polarization: $I' = I\cos^2\theta$.

7. Ray optics:

   (a) General equations:

   $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \qquad R = 2f, \qquad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \tag{22}$$

   (b) Refraction:

   $$n_1\sin\theta_1 = n_2\sin\theta_2, \qquad n = \frac{c}{v}, \qquad c = 3.00 \times 10^8\,\mathrm{m/s}. \tag{23}$$

Equation Sheet for PHYS-205A University Physics

(Last updated: January 16, 2023)
This list will evolve during the semester.

1. General mathematics:

   (a) Units:

$$c = 10^{-2}, \quad m = 10^{-3}, \quad \mu = 10^{-6}, \quad n = 10^{-9}, \quad p = 10^{-12}. \tag{1a}$$

$$d = 10^{2}, \quad k = 10^{3}, \quad M = 10^{6}, \quad G = 10^{9}, \quad T = 10^{12}. \tag{1b}$$

   (b) Geometry of a right triangle:

$$\sin\theta = \frac{\text{opp. to angle}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opp. to angle}}{\text{adj to angle}}, \tag{2a}$$

$$\cos\theta = \frac{\text{adj. to angle}}{\text{hypotenuse}}, \qquad A^2 = A_x^2 + A_y^2. \tag{2b}$$

   (c) Quadratic equation:

$$a\,x^2 + b\,x + c = 0, \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{3}$$

   (d) Calculus:

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad \int x^n dx = \frac{x^{n+1}}{n+1}. \tag{4}$$

2. Kinematic equations:

   (a) Velocity and acceleration:

$$v(t) = \frac{dx}{dt}, \qquad a(t) = \frac{dv}{dt}. \tag{5}$$

   (b) Uniform velocity ($a = 0$):

$$\Delta x = v\Delta t. \tag{6}$$

   (c) Uniform acceleration:

$$v_f = v_i + a\,\Delta t; \qquad \Delta x = v_i\Delta t + \frac{1}{2}\,a\,\Delta t^2; \qquad v_f^2 = v_i^2 + 2\,a\,\Delta x; \tag{7a}$$

$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}; \qquad \Delta x = v_f\Delta t - \frac{1}{2}\,a\,\Delta t^2. \tag{7b}$$

   (d) Time of flight, horizontal range, and maximum height in projectile motion:

$$\Delta T = \frac{2v_0\sin\theta_0}{g}, \qquad R = \frac{v_0^2\sin 2\theta_0}{g}, \qquad H = \frac{v_0^2\sin^2\theta_0}{2g}. \tag{8}$$

   (e) Relative velocity: $\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AG} + \vec{\mathbf{v}}_{GB}$.

3. Forces:

   (a) Newton's law:

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots = m\vec{\mathbf{a}} \tag{9}$$

   (b) Gravitational force:

$$F_G = \frac{Gm_1m_2}{R^2}, \qquad G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}. \tag{10}$$

   (c) Force due to friction:

$$F_\mathrm{f} \begin{cases} \le \mu_s N, & \text{(static case)}, \\ = \mu_k N, & \text{(kinetic case)}. \end{cases} \tag{11}$$

   (d) Circular motion:

$$v = \omega r, \qquad \omega = 2\pi f, \qquad f = \frac{1}{T}, \tag{12}$$

$$a_c = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r = \frac{4\pi^2}{T^2}r \tag{13}$$

   (e) Resistive forces: $R = bv$ (for small speeds) and $R = \frac{1}{2}D\rho A v^2$ (for high speeds).

4. Work and energy:

   (a) Kinetic energy:

$$K = \frac{1}{2}mv^2 \tag{14}$$

   (b) Work done by a force:

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}} \to Fd\cos\theta \tag{15}$$

   (c) Work-kinetic energy theorem:

$$W_1 + W_2 + \ldots = \Delta K \tag{16}$$

   (d) Potential energies:

$$U_g = mgh, \qquad U_s = \frac{1}{2}kx^2. \tag{17}$$

   (e) Mechanical energy:

$$\Delta K + \Delta U_g + \Delta U_s = W_\mathrm{fric} + \ldots. \tag{18}$$

5. Linear momentum:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}, \qquad \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \int_i^f \vec{\mathbf{F}}dt. \tag{19}$$

   (a) Conservation of linear momentum:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} + \ldots = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} + \ldots. \tag{20}$$

   (b) Elastic collision of two objects in one dimensional motion:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}, \tag{21a}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}. \tag{21b}$$

   (c) Center of mass:

$$X_\mathrm{cm} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots} \to \frac{\int x\,dm}{\int dm}. \tag{22}$$

6. Rotational dynamics:

   (a) Kinematic equations:

      i. Constant angular speed ($\alpha = 0$): $\Delta\theta = \omega\Delta t$.

      ii. Constant angular acceleration:

$$\omega_f = \omega_i + \alpha\,\Delta t, \qquad \Delta\theta = \omega_i\Delta t + \frac{1}{2}\,\alpha\,\Delta t^2, \tag{23}$$

$$\omega_f^2 = \omega_i^2 + 2\,\alpha\,\Delta\theta, \qquad \Delta\theta = \omega_f\Delta t - \frac{1}{2}\,\alpha\,\Delta t^2, \qquad \frac{\Delta\theta}{\Delta t} = \frac{\omega_i + \omega_f}{2}. \tag{24}$$

   (b) Rotational inertia (moment of inertia): $I = \int r^2\,dm$.

$$I = \begin{cases} MR^2, & \text{Point mass, distance } R \text{ from axis}, \\ MR^2, & \text{Circular ring, about symmetry axis of ring}, \\ \frac{2}{3}MR^2, & \text{Spherical shell, about diameter}, \\ \frac{1}{2}MR^2, & \text{Solid cylinder, about symmetry axis of cylinder}, \\ \frac{2}{5}MR^2, & \text{Solid sphere, about diameter}. \end{cases} \tag{25a}$$

   (c) Torque:

$$\tau = RF\sin\theta, \qquad \vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}. \tag{26}$$

   (d) Rotational kinetic energy:

$$K_\mathrm{rot} = \frac{1}{2}I\omega^2. \tag{27}$$

   (e) Angular momentum:

$$L = I\omega, \qquad \vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \tag{28}$$

7. Gravitation:

$$\vec{\mathbf{F}} = -\hat{\mathbf{r}}\frac{Gm_1m_2}{r^2}, \qquad G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}. \tag{29}$$

$$U = -\frac{Gm_1m_2}{r}. \tag{30}$$

8. Waves and oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}, \qquad \lambda = \frac{2\pi}{k}, \qquad v = \lambda f. \tag{31}$$

   (a) Oscillations:

$$x = A\cos(\omega t + \delta) \tag{32}$$

   (b) Simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{33}$$

   (c) Mass-spring system:

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{34}$$

9. Fluid dynamics:

   (a) Denisty-mass relation: (Density of water = 10^3 kg/m^3.)

$$m = \rho V. \tag{35}$$

   (b) Pressure:

$$P = \frac{F}{A}. \tag{36}$$

   (c) Pressure in a static fluid:

$$P_2 = P_1 + \rho hg. \tag{37}$$

   1 atm = 1.01×10^5 N/m^2.

   (d) Buoyant force:

$$B = m_\mathrm{dis}g, \tag{38}$$

   where $m_\mathrm{dis} = \rho_\mathrm{dis}V_\mathrm{dis}$ is the mass of the displaced liquid.

   (e) Continuity equation:

$$A_1v_1 = A_2v_2. \tag{39}$$

   (f) Bernoulli's equation:

$$P + \rho hg + \frac{1}{2}\rho v^2 = \mathrm{const} \tag{40}$$