Midterm Exam No. 01 (2025 Spring)

PHYS 510: CLASSICAL MECHANICS

School of Physics and Applied Physics, Southern Illinois University–Carbondale

Date: 2025 Feb 20

1. (**20 points.**) Evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)} \tag{1}$$

of the following functionals, assuming no variation at the end points. Given $a(x)$ is a known function.

$$F[u] = \int_a^b dx\, u(x) \sqrt{1 + a(x)\frac{du}{dx}} \tag{2}$$

2. (**20 points.**) Describe the motion corresponding to the Hamiltonian

$$H(\mathbf{r}, \mathbf{p}) = \frac{p^2}{2m} + \frac{1}{2}k(x^2 - y^2), \tag{3}$$

where $\mathbf{r} = \hat{\mathbf{i}}\, x + \hat{\mathbf{j}}\, y + \hat{\mathbf{k}}\, z$ is position $\mathbf{p}$ is the associated momentum, and m and k are constants. In particular, plot the trajectory of motion for the the initial conditions

$$\mathbf{r}(0) = \hat{\mathbf{i}}\, 0 + \hat{\mathbf{j}}\, R + \hat{\mathbf{k}}\, 0, \tag{4a}$$

$$\mathbf{v}(0) = \hat{\mathbf{i}}\, \omega R + \hat{\mathbf{j}}\, 0 + \hat{\mathbf{k}}\, 0, \tag{4b}$$

where $\omega = \sqrt{k/m}$ and R is a non-zero length.

3. (**20 points.**) Given a time-independent Hamiltonian

$$H = H(x, p) \tag{5}$$

and the corresponding Hamilton equations of motion, show that

$$\frac{dH}{dt} = \alpha, \tag{6}$$

where α is a number. Evaluate α. What is the physical interpretation?

4. (**20 points.**) Given the Lagrangian

$$L_1(z, v) = \frac{1}{2}mv^2 - mgz, \tag{7}$$

find the equation of motion. Next, given another Lagrangian

$$L_2(z, v) = \frac{1}{2}mv^2 - mgz + bvz, \tag{8}$$

find the equation of motion. Analyze and justify.

5. (**20 points.**) A relativistic charged particle of charge q and mass m in the presence of a known electric and magnetic field is described by

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}. \tag{9}$$

(a) Find the Lagrangian for this system, that implies the equation of motion of Eq. (9), to be

$$L(\mathbf{x},\mathbf{v},t) = -mc^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + q\mathbf{v}\cdot\mathbf{A}, \tag{10}$$

using Hamilton's principle of stationary action.

(b) Determine the canonical momentum for this system

(c) Determine the Hamiltonian $H(\mathbf{r},\mathbf{p})$ for this system to be

$$H(\mathbf{x},\mathbf{p},t) = \sqrt{m^2c^4 + (\mathbf{p}-q\mathbf{A})^2\,c^2} + q\phi. \tag{11}$$