# Midterm Exam No. 02 (2025 Spring)

## PHYS 510: CLASSICAL MECHANICS

*School of Physics and Applied Physics, Southern Illinois University–Carbondale*

Date: 2025 Mar 27

1. (**20 points.**) Two masses $m_1$ and $m_2$ are constrained to be on the $x$ axis, but are free to move on the axis. They are connected by a spring of spring constant $k$. Negelect the gravitational force between them. Determine the normal modes of vibration for this system.

2. (**20 points.**) Find the normal modes for the system described by the Lagrangian

$$L(\mathbf{r}, \mathbf{v}) = \frac{1}{2}\mathbf{v}^T \cdot \mathbf{M} \cdot \mathbf{v} - \frac{1}{2}\mathbf{r}^T \cdot \mathbf{K} \cdot \mathbf{r}, \tag{1}$$

   where $\mathbf{r} = (x_1, x_2)$ is a position vector on a plane and $\mathbf{v} = \dot{\mathbf{r}}$ is velocity. Matrices $\mathbf{M}$ and $\mathbf{K}$ are

$$\mathbf{M} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \quad \text{and} \quad \mathbf{K} = \begin{pmatrix} k_1 & k_3 \\ k_3 & k_2 \end{pmatrix}. \tag{2}$$

3. (**20 points.**) Construct the Hamiltonian from the Lagrangian

$$L(x, \dot{x}) = \frac{1}{2}\frac{\dot{x}^2}{(1+ix)} + 16g(1+ix)^2 \tag{3}$$

   and show that the equation of motion can be expressed in the form

$$\ddot{p} = -\alpha p^3. \tag{4}$$

   Find $\alpha$. Here

$$p = \frac{\partial L}{\partial \dot{x}} \tag{5}$$

   is the canonical momentum.

4. (**20 points.**) Four caterpillars, initially at rest at the four corners of a square of sidelength $L$ centered at the origin, start walking clockwise, each caterpillar walking directly toward the one in front of him. If each caterpillar walks with uniform speed $v$, how long does it take for them to meet at the origin? [From the book titled 'Advanced Mathematical Methods for Scientists and Engineers' by C. M. Bender and S. A. Orszag.]