Solution

PHYS-500A (Midterm Exam 02) Fall 2025

Problem 1

Eigenvalues of σ_x are ± 1. A matrix in its own eigenbasis is diagonal with its eigenvalue.

So,

$$\sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Problem 2

$$\frac{d}{dt} \to -i\omega \qquad x(t) \to \tilde{x}(\omega) \qquad \delta(t) \to 1$$

$$\left((-i\omega)^2 + 2\gamma(-i\omega) + \omega_0^2\right)\tilde{x}(\omega) = 1$$

$$\left(-\omega^2 - 2i\gamma\omega + \omega_0^2\right)\tilde{x}(\omega) = 1$$

$$\tilde{x}(\omega) = -\frac{1}{(\omega^2 + 2i\gamma\omega - \omega_0^2)}$$

Problem 3

initial conditions

$x(0-) = 0$

$v(0-) = V_0$.

$$\frac{d^2x}{dt^2} = -V_0\,\delta(t)$$

$$\frac{d^2x}{dt^2} = \begin{cases} 0, & t<0, \\ 0, & 0<t. \end{cases}$$

$$x(t) = \begin{cases} At + B, & t<0, \\ ct + D, & 0<t. \end{cases}$$

$$v(t) = \begin{cases} A, & t < 0, \\ C, & 0 < t. \end{cases}$$

Using initial conditions we have. $B = 0$ and $A = V_0$.

Thus, we have.

$$x(t) = \begin{cases} V_0 t, & t < 0, \\ Ct + D, & 0 < t, \end{cases}$$

$$v(t) = \begin{cases} V_0, & t < 0, \\ C, & 0 < t. \end{cases}$$

Continuity conditions are

(i) $$v(t)\Big|_{0-}^{0+} = -V_0$$ → momentum-impulse conservation.

(ii) $$x(t)\Big|_{0-}^{0+} = 0$$

Thus, we have.

$$C - V_0 = -V_0 \quad \Rightarrow \quad C = 0$$

and

$$0 + D = 0 \quad \Rightarrow \quad D = 0$$

Thus, the mass comes to a stop at the origin.

$$x(t) = \begin{cases} V_0 t, & t < 0, \\ 0, & 0 < t, \end{cases}$$

$$v(t) = \begin{cases} V_0, & t < 0, \\ 0, & 0 < t. \end{cases}$$

Problem 4

$$\int_{-1}^{1} dx \frac{(5x^3-3x)}{\sqrt{1+t^2-2tx}} = 2\int_{-1}^{+1} dx \frac{P_3(x)}{\sqrt{1+t^2-2tx}}$$

$$= 2\int_{-1}^{+1} dx\, P_3(x) \sum_{l=0}^{\infty} t^l P_l(x)$$

$$= 2\sum_{l=0}^{\infty} t^l \int_{-1}^{+1} dx\, P_3(x) P_l(x)$$

$$= 2\sum_{l=0}^{\infty} t^l \frac{2}{2(3)+1} \delta_{3l}.$$

$$= 2\, t^3 \frac{2}{7}$$

$$= \frac{4}{7} t^3$$